The effect of entropy order in image alignment by maximum mutual information criterion

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Introduction

Image alignment is used in a wide variety of fields, particularly in medicine for the alignment of positron emission, magnetic resonance and computed tomography images [1]. The task of image alignment comes down to the fact that points corresponding to identical elements of the scene structure are aligned by a spatial geometric transformation of two or more images formed by sensor. Intensity-based alignment involves only the estimation of the numerical value of a similarity measure. Such measures include Shannon, Renyi and Tsallis mutual information (MI) [2].

Problem statement

When several images are combined, each subsequent image is attached to an image, which can conventionally be called a reference image. Then the other images will have spatial deformations in relation to the reference image. These parameters need to be determined. If another image is selected as the reference image, then the combination of images is still performed in pairs. Therefore we will consider the alignment of the two images: the reference image $\mathbf{Z}^{(1)}$ and the deformed image $\mathbf{Z}^{(2)}$. To find deformation parameters $\bar{\alpha}$, we have to define some deformation model, which allows combining $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ according to some optimality criterion. We use a similarity model as our deformation model. The work uses a stochastic matching procedure. It allows one to determine the optimal $\bar{\alpha}^{*}$ parameters of geometric transformation for a given similarity measure.

We investigate the Shannon, Renyi and Tsallis MIs as target functions [2]. These target functions depend on the entropy order. The influence of this parameter on the efficiency of matching procedures synthesized based on these measures is practically unexplored. Therefore, the purpose of this paper is to investigate the effect of the entropy order of Renyi and Tsallis on the convergence speed of the stochastic procedure, including the additive noise conditions.

Experimental results

The real and simulated images were used in the study. Figure 1a shows the shift dependences of the normalized Shannon MI (curve 1) and Renyi at three entropy order values q = 0.5 (curve 2), q = 1.1 (curve 3) and q = 6 (curve 4). We can see that at different entropy orders, the results differ slightly from each other and Shannon's MI one. For a priori prediction and comparison of the potential and average convergence rate of the parameter estimate vector the mean maximum slope of the functionals and were estimated. The corresponding dependencies are shown in Figure 1b and 1c. An example of the corresponding dependencies is shown in Figure 1d.

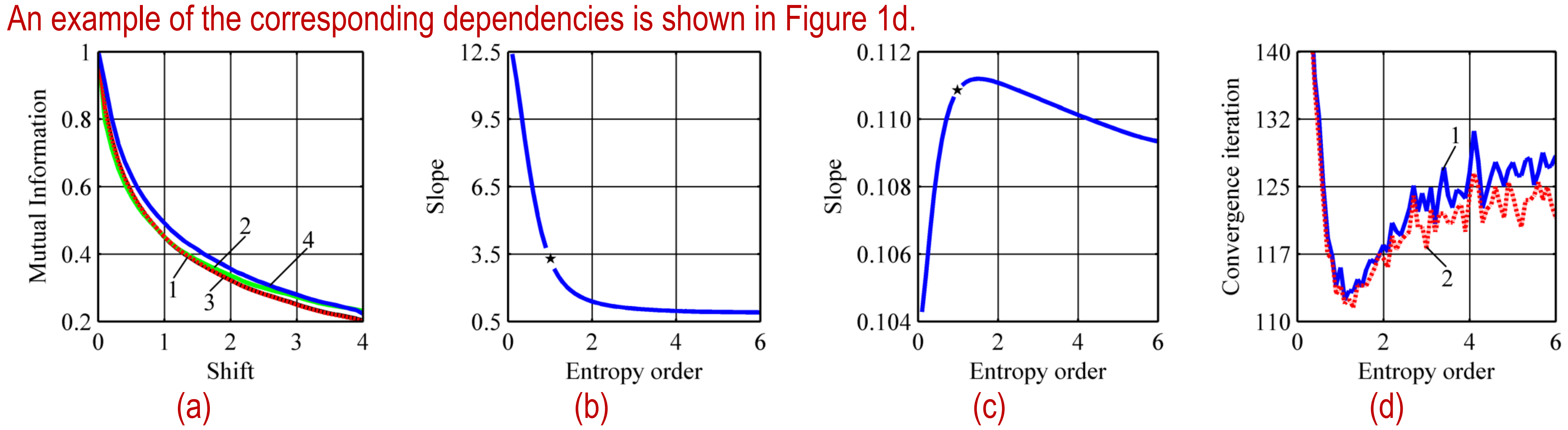

Figure 1: (a) Shift dependences; (b) Maximal slope; (c) Average slope; (d) Convergence rate

The shift dependences of the Tsallis MI at the same range of entropy order as for the Renyi MI are shown in Figure 2a. In the figure, curve 1 corresponds to the Shannon MI, the numbers of other curves correspond to the same values of the entropy order as when considering the Renyi MI. The figure shows that the effect of the order of entropy on the Tsallis MI is significantly stronger than on the Renyi MI. With a decrease in the entropy order, the potential working range increases, but the potential convergence rate of the parameters decreases.

The shift dependences of the Tsallis MI at the same range of entropy order as for the Renyi MI are shown in Figure 2a. The dependencies of the maximum and average slope are shown in Figure 2b and Figure 2c, respectively.The results of the experimental checking using the stochastic procedure with the same images and deformation parameters as for the Renyi MI are shown in Figure 2d. The figure shows that the highest convergence rate of the deformation parameter estimates is achieved in the same entropy order range as the maximum average slope of the MI

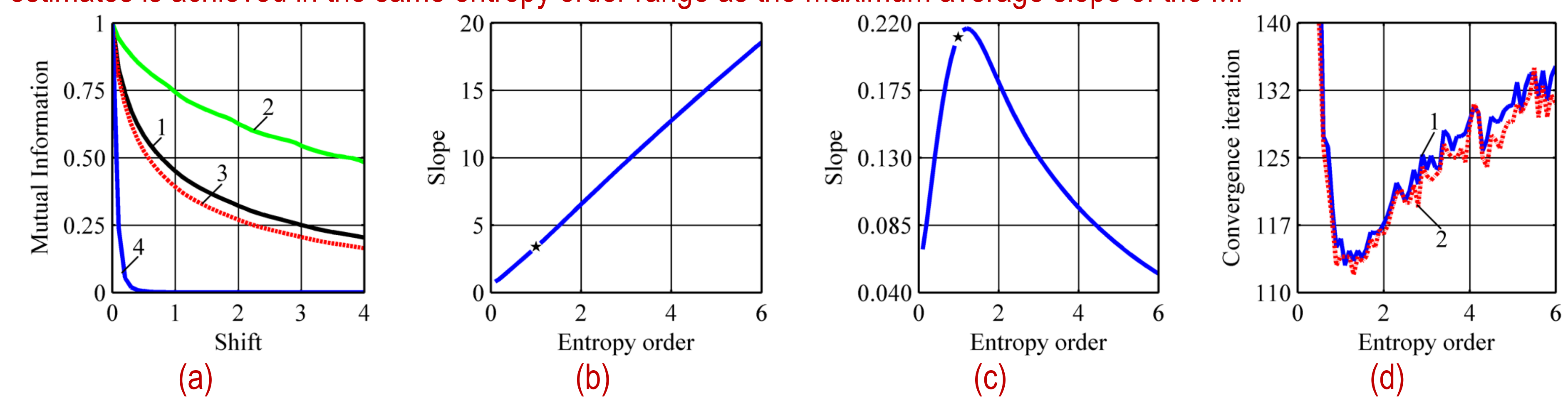

Figure 2: (a) Shift dependences; (b) Maximal slope; (c) Average slope; (d) Convergence rate

The effect of entropy order on the performance of relay-based stochastic image alignment procedures under noise conditions has also been studied. For the result below, the signal-to-noise ratio by variance was 3.7.

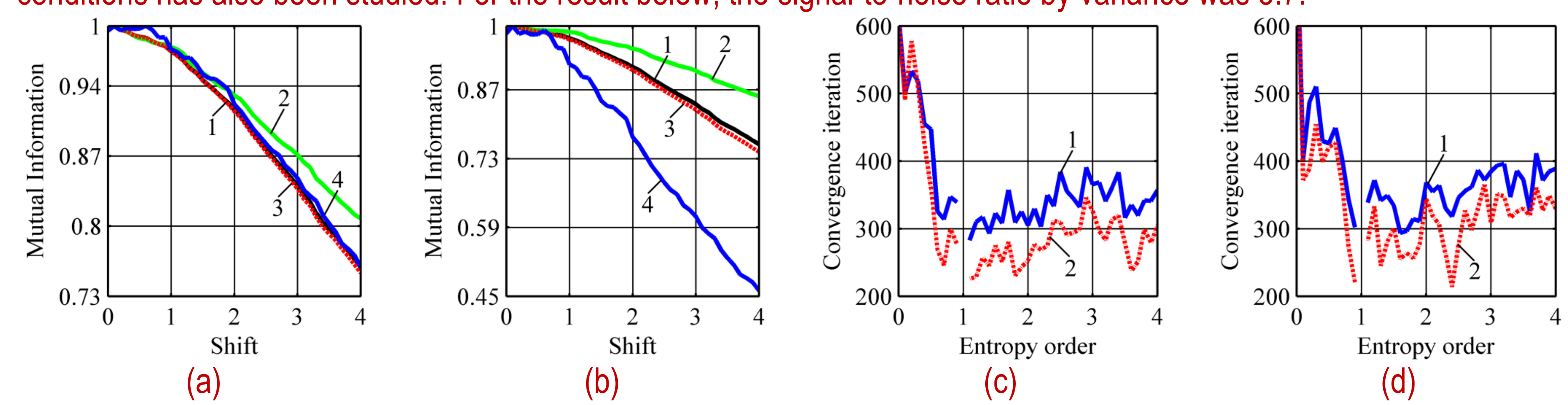

Figure 3: (a) Renyi MI with noise; (b) Tsallis MI with noise; (c) Renyi convergence rate; (d) Tsallis convergence rate

Conclusion

The research has shown that in the design of stochastic image alignment procedures based on information measures, the choice of entropy order in Renyi and Tsallis MIs affects the convergence rate of the alignment parameter estimates. A priori analysis of the MI function slope makes it possible to find values of the entropy order at which the maximum convergence rate is achieved. Thus, it is possible to select a priori the image similarity measure most suitable for the particular application problem before synthesizing the alignment procedure. We found that the optimum entropy order for the Renyi MI was approximately in the range of 0.9 to 1.3. And for Tsallis MI it was between 1.1 and 1.4. The relay stochastic procedures synthesized based on Renyi and Tsallis MIs in experimental studies confirmed these conclusions. It is also shown that additive noise has little effect on the optimal entropy order. But the noise case, the alignment procedures with optimal entropy order showed a higher convergence rate than the Shannon MI procedure.

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References

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