Sudden death of entanglement in a thermal three-qubit Tavis-Cummings model

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The model and its exact solution

We consider a system of three identical qubits (two-level atoms) resonantly interacting with the mode of the quantum electromagnetic field of an ideal microwave resonator.

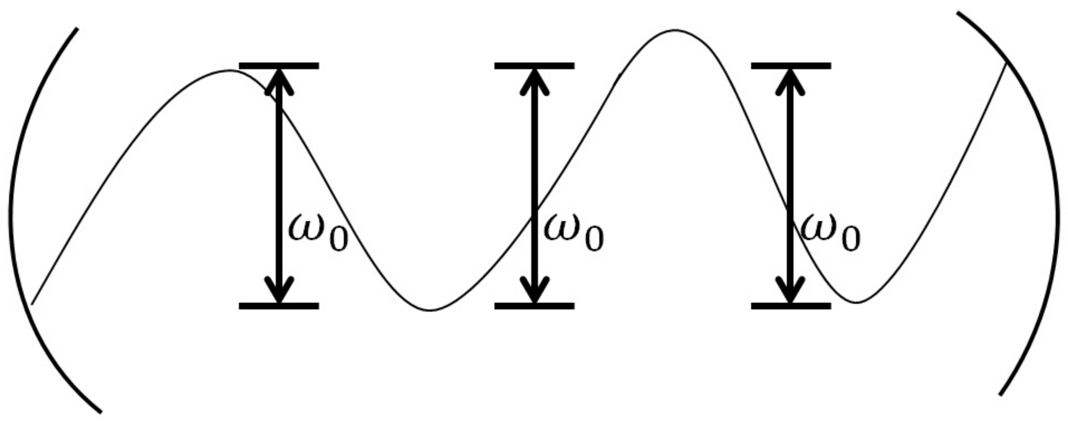

Figure: 1) A diagram of three two-level atoms in a resonator.

The Hamiltonian of the interaction of such a system in the dipole approximation and the rotating wave approximation is

$$\hat{H} = \hbar\gamma \sum_{l=1}^{3} (\hat{\eta}^{+} \hat{R}_l^{-} + \hat{R}_l^{+} \hat{\eta}) \tag{1}$$

where $\hat{R}_l^{+}$ and $\hat{R}_l^{-}$ are the rasing and the lowering operators in the l-th qubit ($l = 1, 2, 3$), $\hat{\eta}^{+}$ and $\hat{\eta}$ are the creation and annihilation of resonator photons and γ is the qubit-photon coupling.

As the initial state of the resonator field, we choose a thermal state with a density matrix of the form:

$$_F(0) = \sum_m W_m |m >< m| \tag{2}$$

There are weight coefficients

$$W_m = \frac{\overline{m}^m}{(1+\overline{m})^{m+1}}, \; \overline{m} = (exp[\hbar\omega/kT] - 1)^{-1} \tag{3}$$

where $\bar{n}$ – average number of thermal photons.

We have found an exact solution to the Schrodinger time equation:

$$i\hbar \frac{\partial}{\partial t} |\Phi_n(t) >_{Q_1Q_2Q_3F} = \hat{H} |\Phi_n(t) >_{Q_1Q_2Q_3F} \tag{4}$$

Selection of initial states and calculation of negativity

Biseparable states are selected as initial states:

$$|\Phi(0) >_{A_1A_2A_3} = cos(\alpha)|ex, ex, gr > + sin(\alpha)|ex, gr, ex > \tag{5}$$

$$|\Phi(0) >_{A_1A_2A_3} = cos(\alpha)|gr, ex, gr > + sin(\alpha)|gr, gr, ex > \tag{6}$$

and truly entangled W-type states

$$|\Phi(0) >_{A_1A_2A_3} = d|ex, ex, gr > + f|ex, gr, ex > + g|gr, ex, ex > \tag{7}$$

where $|ex >_l$ is the excited state and $|gr >_l$ ground state of l-th qubit.

To calculate the negativity of two qubits, it is necessary to calculate a reduced two-qubit density matrix, which in our case has the form:

$$W_{A_iA_j}^T(t) = \begin{pmatrix} W^{(11)}(t) & 0 & 0 & (W^{(23)}(t))^* \\ 0 & W^{(22)}(t) & 0 & 0 \\ 0 & 0 & W^{(33)}(t) & 0 \\ W^{(23)}(t) & 0 & 0 & W^{(44)}(t) \end{pmatrix} \tag{8}$$

For a system of two qubits A_i and A_j, we define the Peres-Horodetsky parameter or negativity in a standard way

$$\varepsilon_{ij} = -2 \sum_l (\nu_{ij}^{-})_l \tag{9}$$

where $(\nu_{ij}^{-})_l$– negative eigenvalues of the density matrix $W_{A_iA_j}^T$.

Given the form of the density matrix (5), we have the negativity:

$$\varepsilon_{ij} = \sqrt{(W^{(11)} - W^{(44)})^2 + |W^{(23)}|^2} - W^{(11)} - W^{(44)} \tag{10}$$

Discussion of the results

The results of computer simulation of the time dependence of negatives:

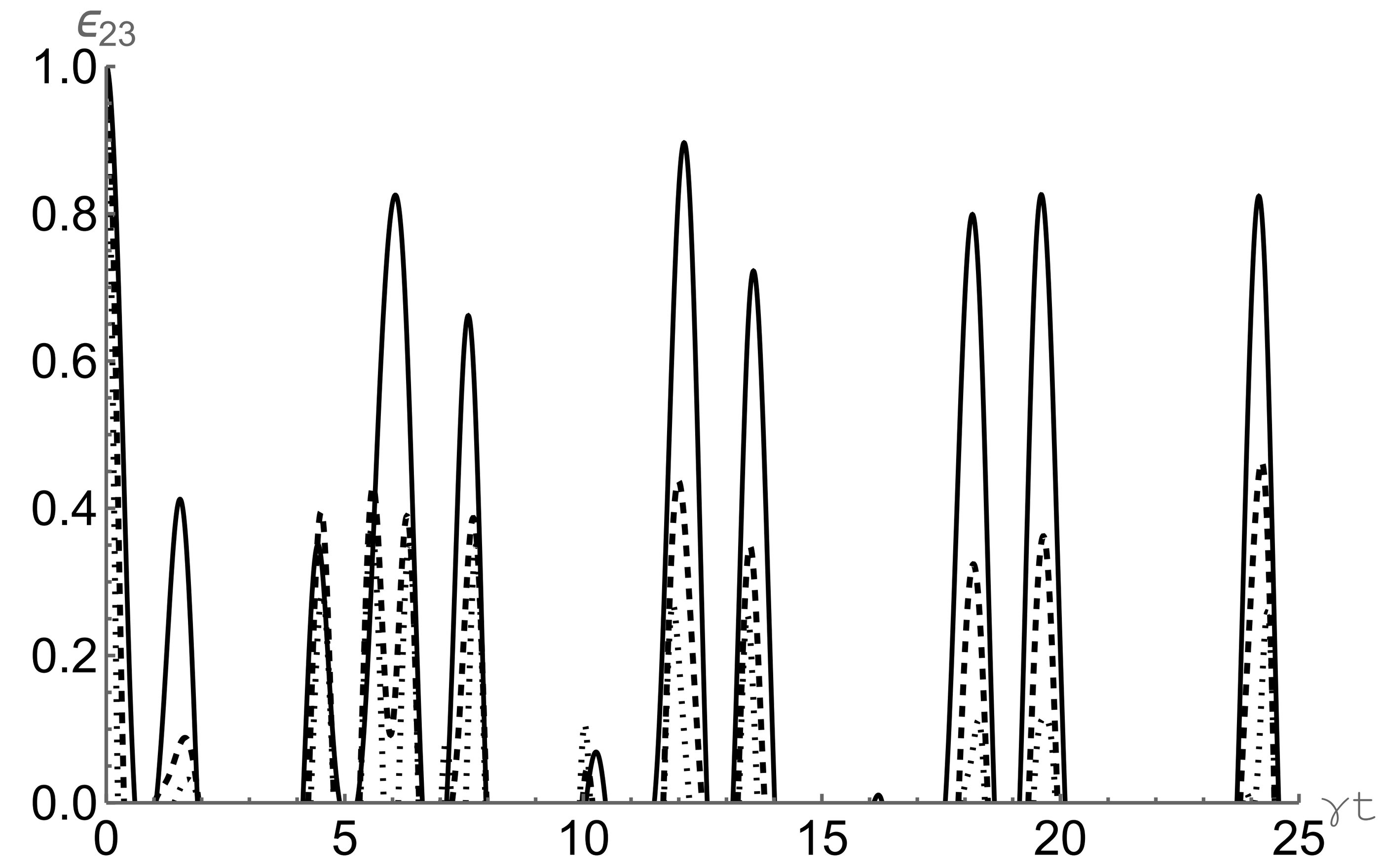

Figure: 2) The entanglement $\varepsilon_{23}(t)$ as a function of a dimensionless time γt for biseparable state (5) for $\alpha = \frac{\pi}{4}$. The intensity of the cavity field for graph is $< m >= 0.1$ (solid), $< m >= 1$ (dashed) and $< m >= 3$ (dotted).

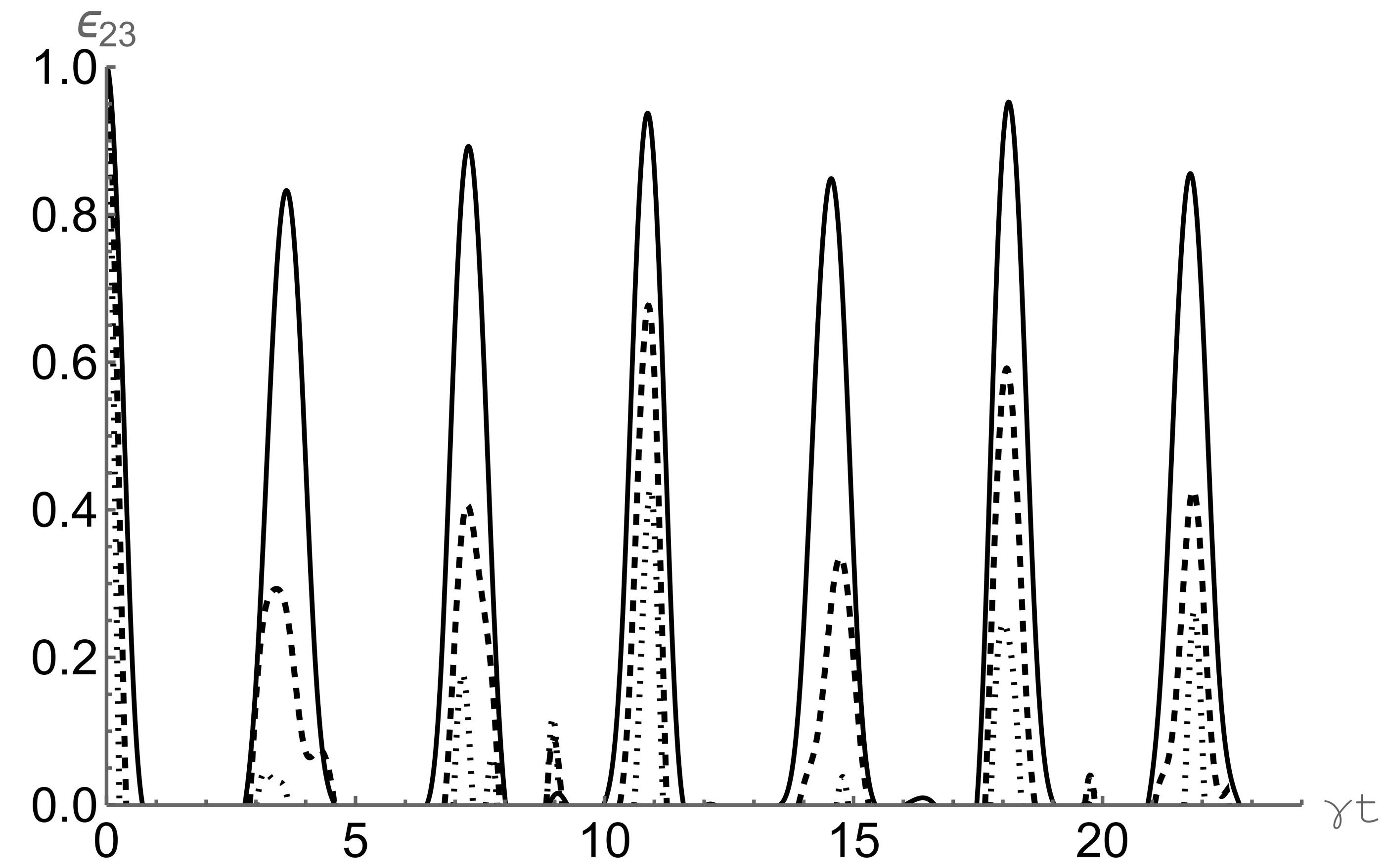

Figure: 3) The entanglement $\varepsilon_{23}(t)$ as a function of a dimensionless time γt for biseparable state (6) for $\alpha = \frac{\pi}{4}$. The intensity of the cavity field for graph is $< m >= 0.1$ (solid), $< m >= 1$ (dashed) and $< m >= 3$ (dotted).

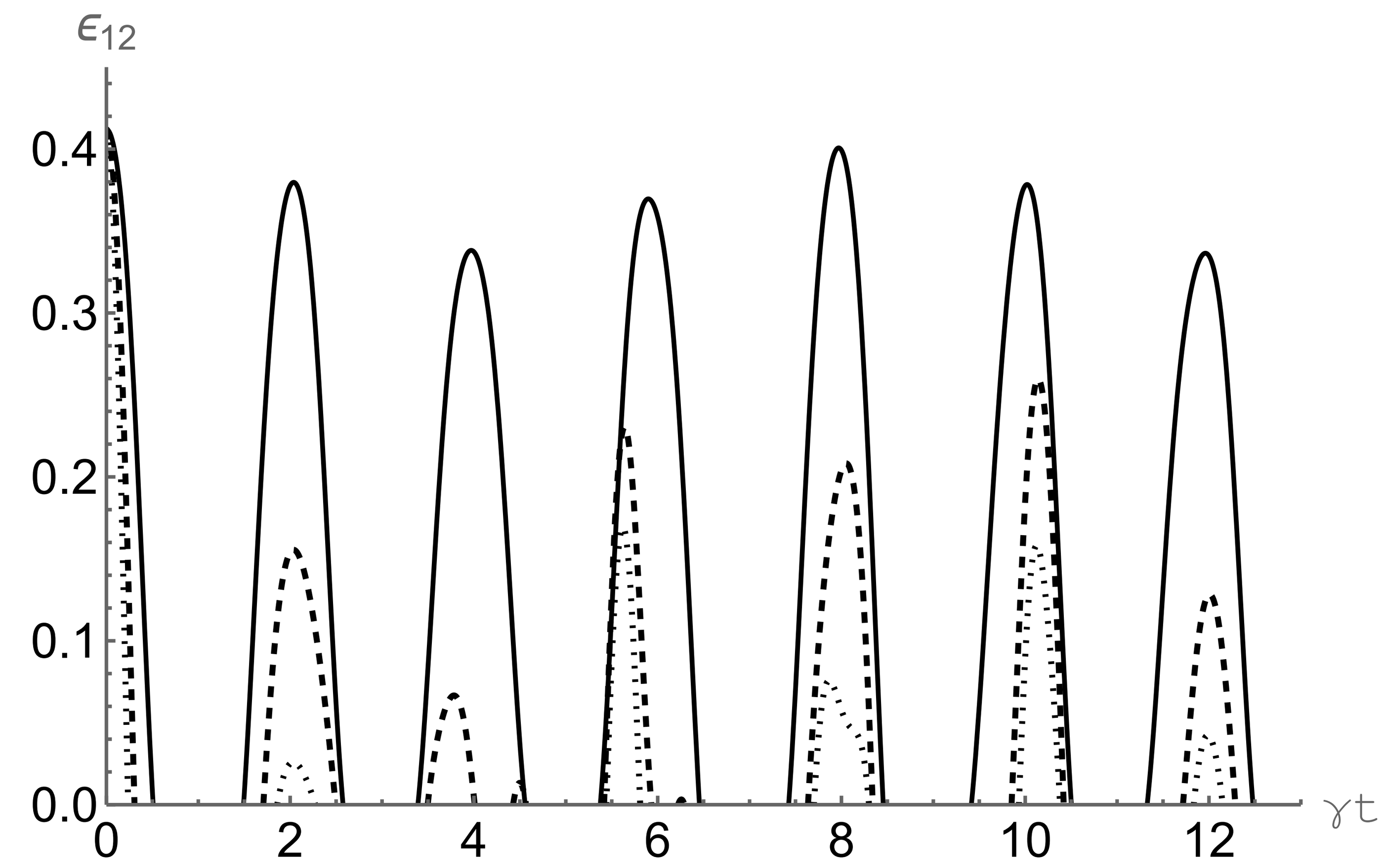

Figure: 4) The entanglement $\varepsilon_{12}(t)$ as a function of a dimensionless time γt for W-type entangled -state (7) with $d = f = g = \frac{1}{\sqrt{3}}$. The intensity of the cavity field $< m >= 0.1$ (solid), $< m >= 1$ (dashed) and $< m >= 2$ (dotted).

In the present context, the model consisting of three identical qubits interacting with a one-mode thermal field of lossless resonator was investigated. The exact solution of the quantum Liouville equation for the model under consideration was found. The biseparable and genuinely entangled W-type states of qubits were in the focus of our attention. The exact solution was used to investigate the qubit-qubit pair entanglement induced by the thermal field. The results showed that for three qubits interacting with the thermal field the sudden death of entanglement takes place for all considered initial qubits states and various values of the average thermal photon numbers. As well-known for two-qubit system this effect occurs only for intensive thermal field. The study of mechanisms that suppress the sudden death of entanglement, such as dipole-dipole interaction, Kerr nonlinearity, detuning, etc., will be the subject of our next article.