Hall effect near a sharp focus of cylindrical vector beams with negative order

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Introduction

– In optics, cylindrical vector beams (CVB) are well known [1], including the high-order beams.

– To confirm the theoretic findings, we performed a numerical simulation using Richards-Wolf formula

– We have investigated the behavior of the intensity, components of the Poynting vector **P** = Re[**E**×**H***] and spin angular momentum (SAM) **S** = Im[**E***×**E**] when focusing a high-order cylindrical vector beams by aplanatic lens with a numerical aperture NA = 0.95.

– In the sharp focus of the nth-order CVB, the intensity distribution has 2(*n* – 1) peaks.

– Areas with reverse energy flows could occur in the focal plane

– There are 4(*n* – 1) areas with different rotation direction of the polarization vector

– In the areas where before the focus (*z* < 0) the SAM was negative (*S*3 < 0), after the focus (*z* > 0) it becomes positive (*S*3 > 0), and vice versa.

Richards-Wolf formulae

$$\mathbf{U}(\rho,\psi,z)=-\frac{if}{\lambda}\int_0^{\theta_0}\int_0^{2\pi}B(\theta,\varphi)T(\theta)\mathbf{P}(\theta,\varphi)\exp\{ik[\rho\sin\theta\cos(\varphi-\psi)+z\cos\theta]\}\sin\theta\,d\theta\,d\varphi,$$

$$\mathbf{P}(\theta,\varphi)=\begin{bmatrix}1+\cos^2\varphi(\cos\theta-1)\\ \sin\varphi\cos\varphi(\cos\theta-1)\\ -\sin\theta\cos\varphi\end{bmatrix}a(\theta,\varphi)+\begin{bmatrix}\sin\varphi\cos\varphi(\cos\theta-1)\\ 1+\sin^2\varphi(\cos\theta-1)\\ -\sin\theta\sin\varphi\end{bmatrix}b(\theta,\varphi),$$

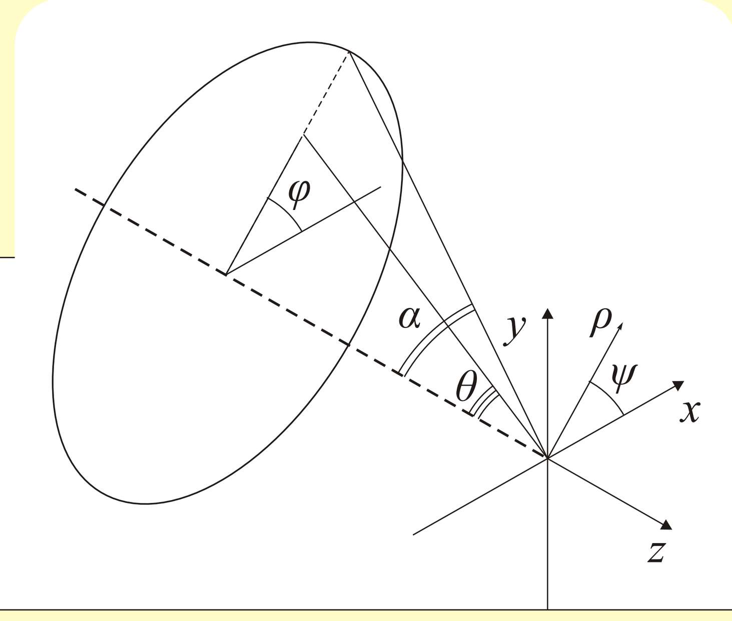
$$E_n(\varphi)=\begin{pmatrix}a(\theta,\varphi)\\ b(\theta,\varphi)\end{pmatrix}=\begin{pmatrix}\cos n\varphi\\ \sin n\varphi\end{pmatrix},\quad H_n(\varphi)=\begin{pmatrix}a(\theta,\varphi)\\ b(\theta,\varphi)\end{pmatrix}=\begin{pmatrix}-\sin n\varphi\\ \cos n\varphi\end{pmatrix}.$$

$$E_x(r,\varphi)=i^{n-1}\left[\cos(n\varphi)I_{0,n}+\cos((n-2)\varphi)I_{2,n-2}\right],\qquad H_x(r,\varphi)=-i^{n-1}\left[\sin(n\varphi)I_{0,n}+\sin((n-2)\varphi)I_{2,n-2}\right],$$

$$E_y(r,\varphi)=i^{n-1}\left[\sin(n\varphi)I_{0,n}-\sin((n-2)\varphi)I_{2,n-2}\right],\qquad H_y(r,\varphi)=-i^{n-1}\left[-\cos(n\varphi)I_{0,n}+\cos((n-2)\varphi)I_{2,n-2}\right],$$

$$E_z(r,\varphi)=2i^n\cos((n-1)\varphi)I_{1,n-1},\qquad H_z(r,\varphi)=-2i^n\sin((n-1)\varphi)I_{1,n-1}.$$

$$I_{\nu,\mu}=\left(\frac{4\pi f}{\lambda}\right)\int_0^{\theta_0}\sin^{\nu+1}\left(\frac{\theta}{2}\right)\cos^{3-\nu}\left(\frac{\theta}{2}\right)T(\theta)A(\theta)e^{ikz\cos\theta}J_\mu(x)d\theta,$$



$$SAM_z=s_3=2\,\mathrm{Im}(E_x^*E_y)$$

$$SAM_z\big|_{z=0}=0\qquad SAM_z\big|_{e^{ikz\cos\theta}\approx 1+ikz\cos\theta}=2kz\sin[(2m-2)\varphi][Ir_{0,m}Ii_{2,m-2}-Ir_{2,m-2}Ii_{0,m}]$$

$$Ir_{\nu,\mu}=\left(\frac{\pi f}{\lambda}\right)\int_0^{\theta_0}\sin^{\nu+1}\left(\frac{\theta}{2}\right)\cos^{3-\nu}\left(\frac{\theta}{2}\right)T(\theta)A(\theta)J_\mu(x)d\theta,\qquad Ii_{\nu,\mu}=\left(\frac{\pi f}{\lambda}\right)\int_0^{\theta_0}\sin^{\nu+1}\left(\frac{\theta}{2}\right)\cos^{3-\nu}\left(\frac{\theta}{2}\right)T(\theta)A(\theta)\cos\theta J_\mu(x)d\theta.$$

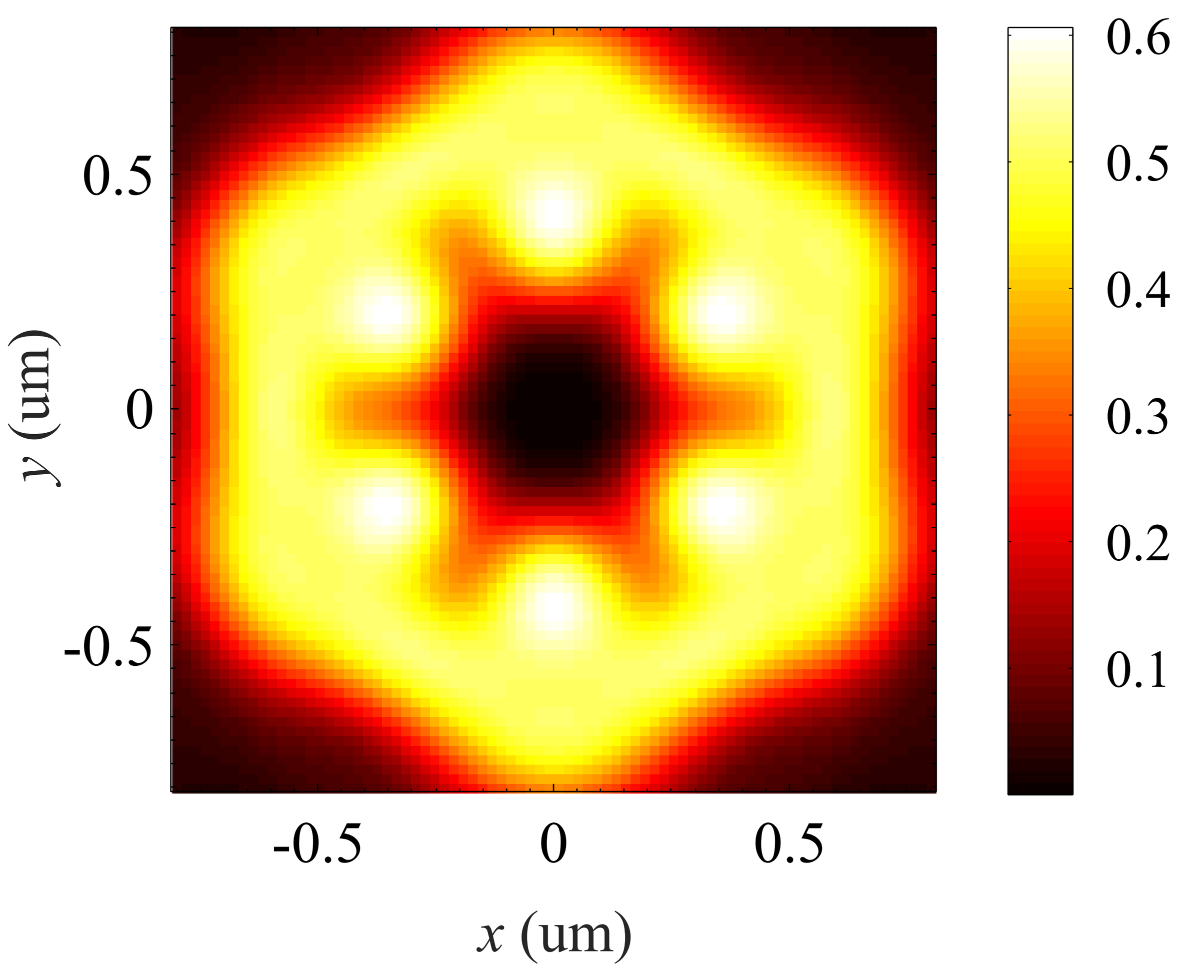
Conclusions

– The tight focusing of high-order cylindrical vector beams was investigated numerically and theoretically

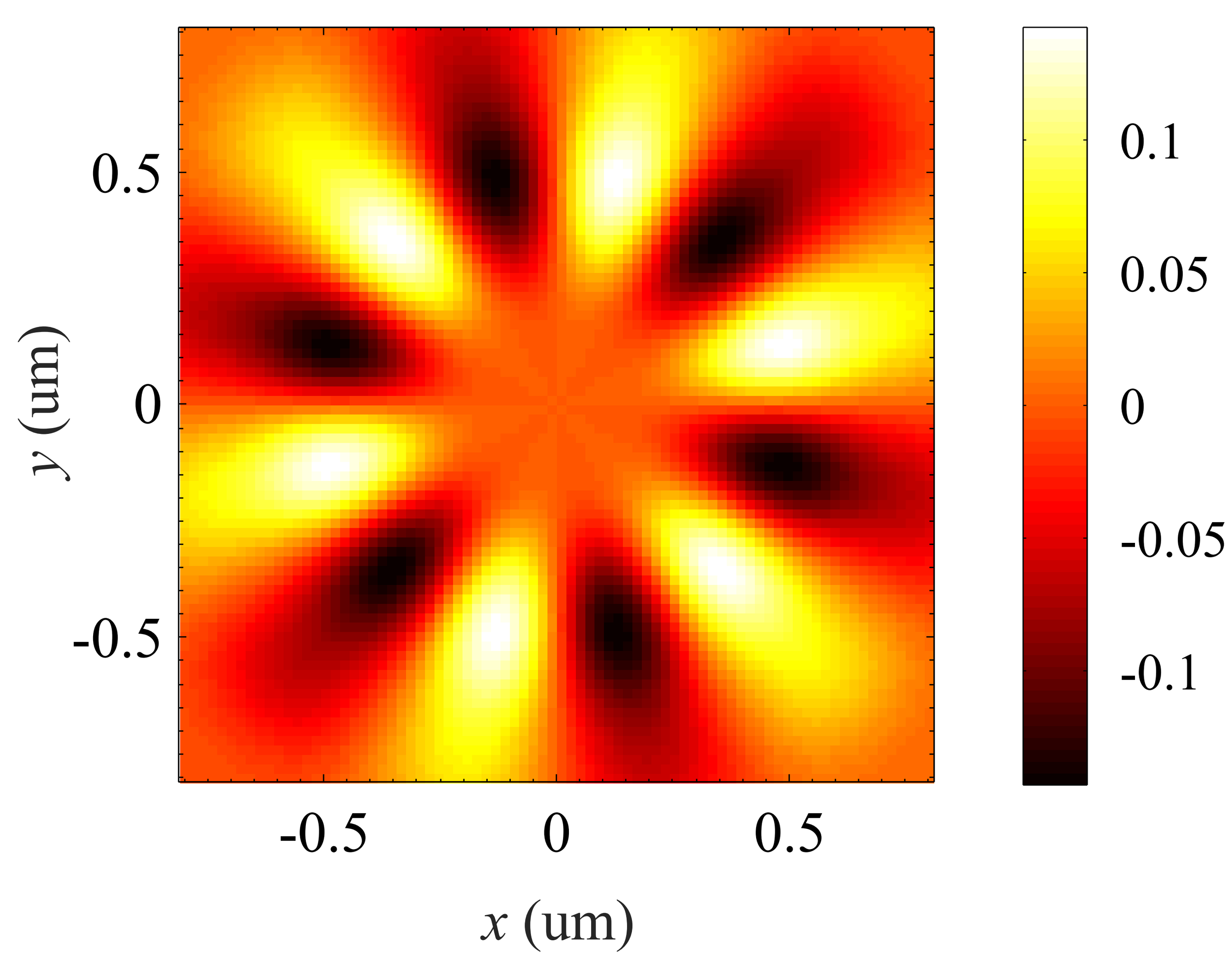
– It was shown that near the focal plane of the CVB, for instance, at a distance of wavelength before and beyond the focus, 4(n – 1) local subwavelength areas are generated, where the polarization vector is rotating in each point.

– For the order of the beam equals to unity (radial polarization) there is no polarization conversion

Orbital and spin energy flows in tight focus of optical vortex



Distribution of intensity of a sharply focused cylindrical beam of the order n=-2 after the focus at the distance z = λ.



Distribution of longitudinal component of the SAM vector of a sharply focused cylindrical beam of the order n=-2 after the focus at the distance z = λ.

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References

[1] Zhan, Q. Cylindrical vector beams: from mathematical concepts to applications. Adv. Opt. Photon. 2009, 1, 1–57.
[2] B. Richards and E. Wolf, "Electromagnetic Diffraction in Optical Systems. II. Structure of the Image Field in an Aplanatic System," Proc. R. Soc. A Math. Phys. Eng. Sci., vol. 253(1274), pp. 358–379, 1959.
[3] A. Y. Bekshaev, "Subwavelength particles in an inhomogeneous light field: optical forces associated with the spin and orbital energy flows," J. Opt. 15, 044004 (2013).
[4] V. V. Kotlyar, S. S. Stafeev, and A. G. Nalimov "Energy backflow in the focus of a light beam with phase/polarization singularity" Phys Rev A, vol. 99, pp. 33840 (2019)