Stability of structured Laguerre-Gauss beams to astigmatic transformation

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Beam model

The simplest way to convert standard Laguerre-Gaussian (LG) beams into structured beams without losing their structural stability is to represent such a beam in terms of the sum of Hermite-Gaussian (HG) modes:

$$LG_{n,\ell}(\mathbf{r})=\frac{(-1)^n}{2^{2n+3\ell/2}n!}\sum_{k=0}^{2n+\ell}\left\{(-2i)^k P_k^{(n+\ell-k,n-k)}(0)\right\}HG_{2n+\ell-k,k}(\mathbf{r}), \quad (1)$$

where $\mathbf{r}=(x,y)$, $P_k^{(n+\ell-k,n-k)}(\cdot)$ Jacobi polynomials, n and ℓ are the radial and azimuth number of LG modes, $HG_{2n+\ell-k,k}(\mathbf{r})$, complex amplitude of the HG beam. Each HG mode in the wave composition (1) can be considered as a separate degree of freedom capable of supporting large information. flows. To take into account the excitation of each HG mode, it is convenient to use a two-parameter perturbation function: $\varepsilon_k(\varepsilon,\theta)=1+\varepsilon e^{ik\theta}$, which translates the standard LG beam into structured beams:

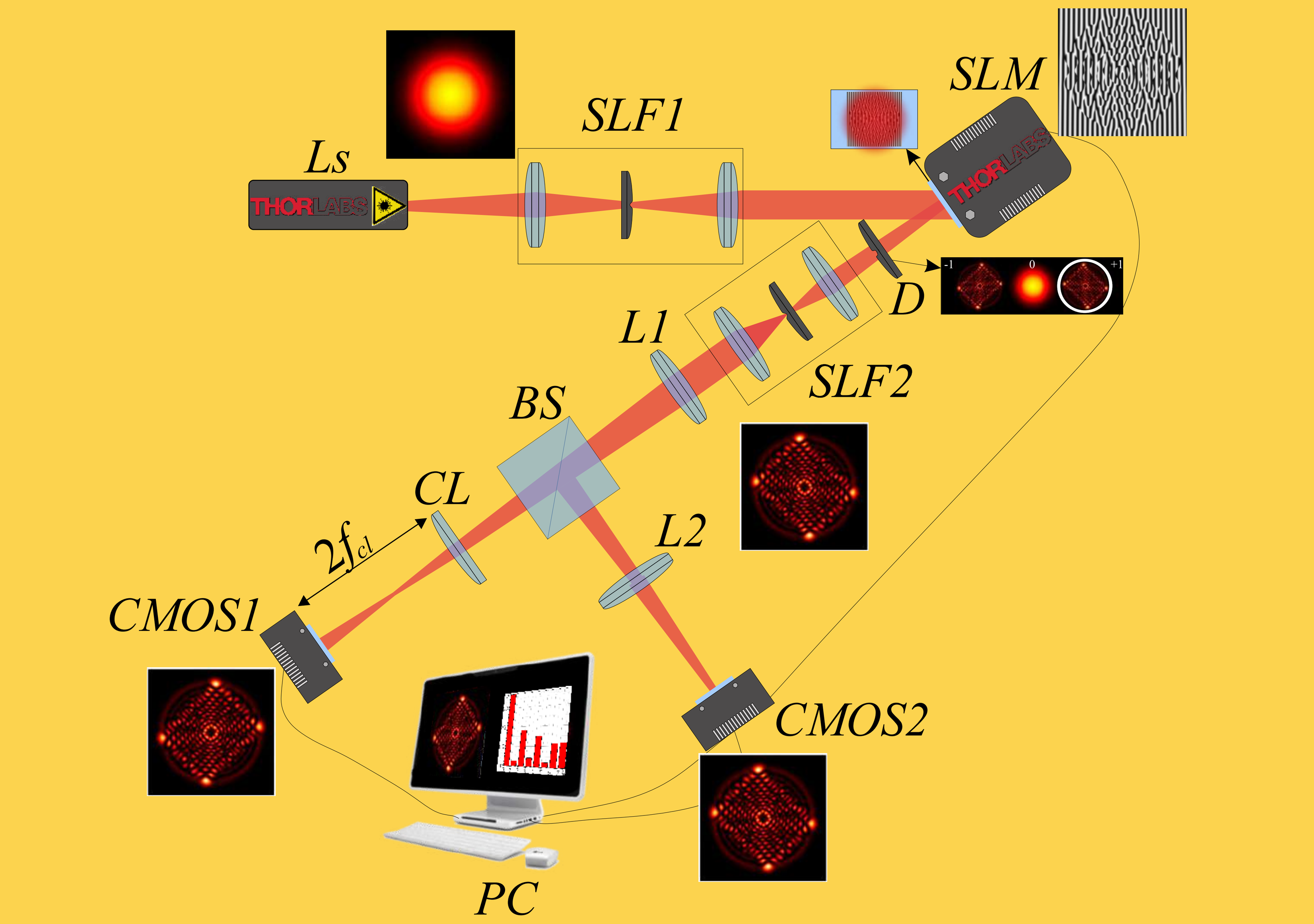
$$\mathrm{sLG}_{n,\ell}(\mathbf{r},\varepsilon,\theta)=\frac{(-1)^n}{2^{2n+3\ell/2}n!}\sum_{k=0}^{2n+\ell}\left\{(2i)^k P_k^{(n+\ell-k,n-k)}(0)\varepsilon_k(\varepsilon,\theta)\right\}\mathrm{HG}_{2n+\ell-k,k}(\mathbf{r}). \quad (2)$$

Here ε and θ represent the amplitude and phase parameters, respectively. Such a transformation is easy to implement in practice using a spatial light modulator. This new family of paraxial beams has a number of unexpected properties. First of all, expression (2) corresponds to the sum of two standard Hermite-Laguerre-Gaussian hybrid beams with different control parameters:

$$sLG(\mathbf{r}|\varepsilon,\theta)=\frac{(-1)^n}{2^{n+\ell}n!}\left\{HLG_{n+\ell,n}(\mathbf{r}|\pi/4)+\varepsilon(-1)^{n+\ell}\times\exp\left(i(2n+\ell)\Theta\right)HLG_{n,n+\ell}\left(\mathbf{R}_{-\pi/4}\cdot\mathbf{r}|\Theta\right),\right. \quad (3)$$

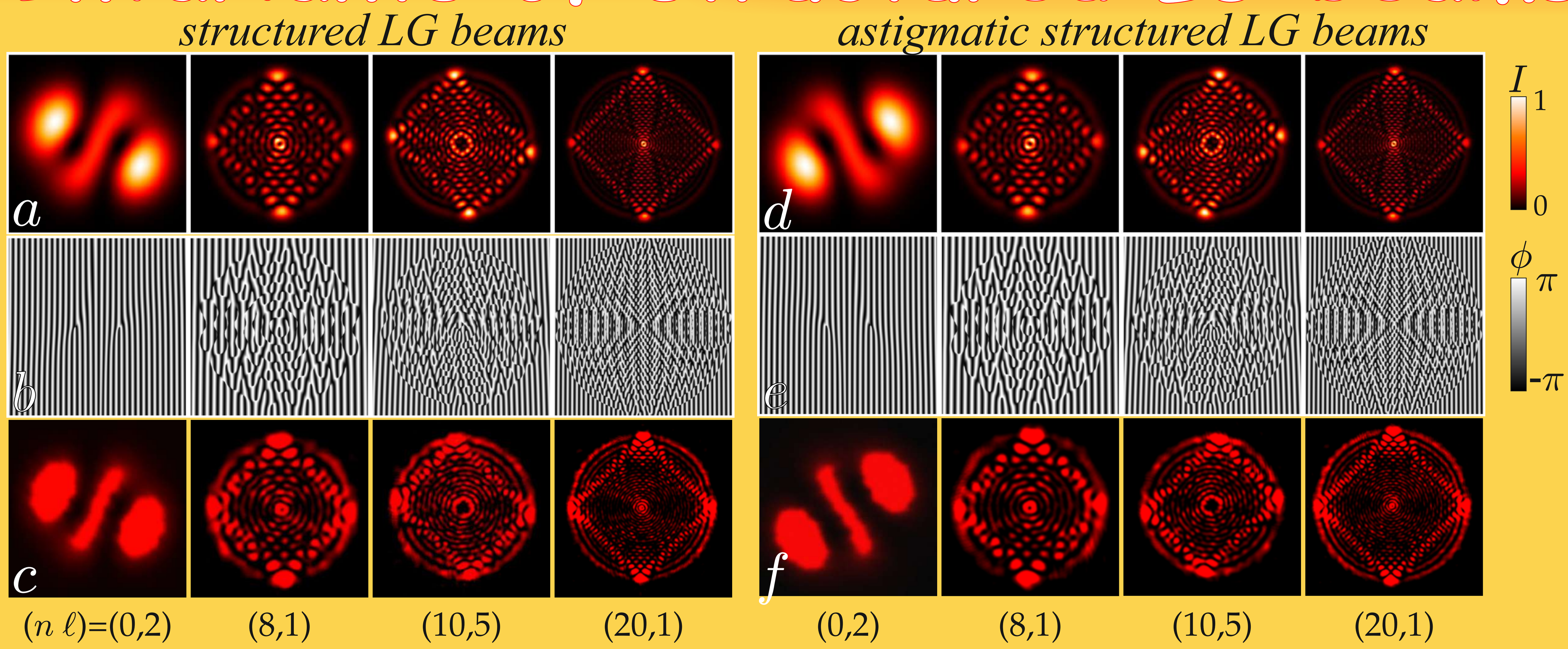
where $\Theta=\theta/2-\pi/4$ is the control parameter of the Hermite-Laguerre-Gaussian beam, $\mathbf{R}_\alpha$ represents 2x2 rotation matrix at angle α. It is important to note that the variation of the control parameter causes fast oscillations of the orbital angular momentum (OAM), which corresponds to fast transitions of the structural LG beam to new structurally stable states. Nevertheless, despite the fast oscillations of the OAM due to internal perturbations, the structured beam has a number of invariants in the form of the conservation of the topological charge modulus. In the presented work, we touch upon the problem of invariants of a structural beam subjected to external perturbations in the form of an astigmatic transformation, for example, when the beam passes through a cylindrical lens.

Experimental setup



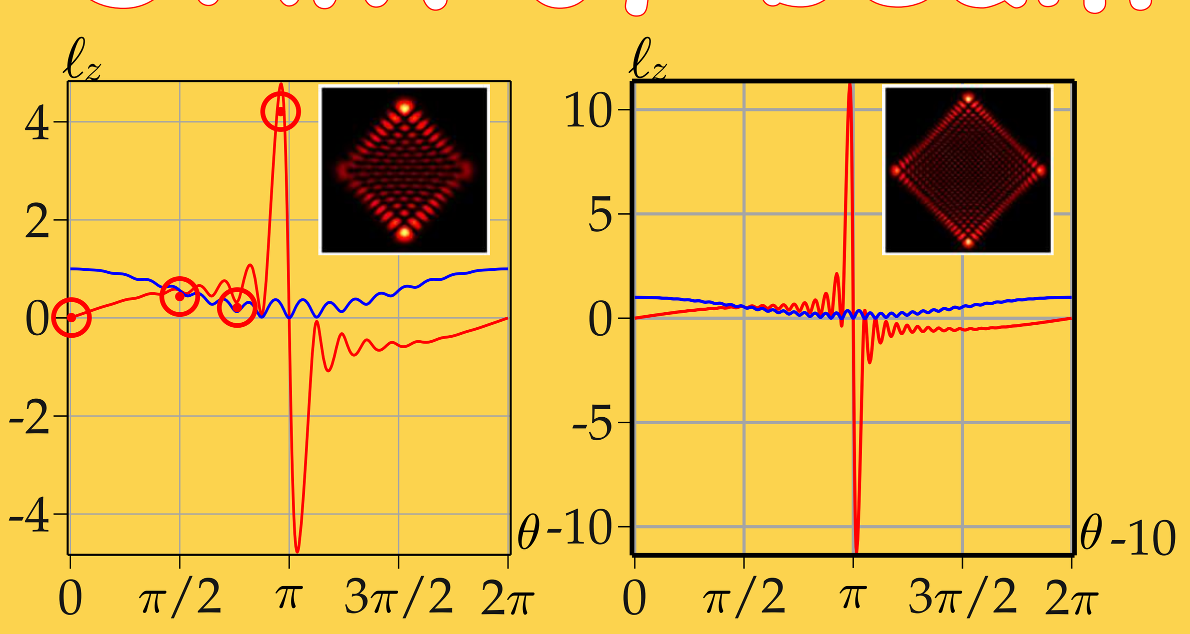
Experimental setup: laser 0.633 μm, SLF1,2 – spatial light filter, L1,2 - lens, with focal length about f =20*cm*, CL – cylindrical lens (f=25*cm*), BS – beam splitter, SLM1 - spatial light modulator, CMOS1,2 – photodetector, PC - computer.

Invariants of structured LG beams



During the experiment, we obtained the intensity distributions of a structured LG beam at the focus of a spherical lens L2 (using a CMOS1 photodetector) and at the double focus of a cylindrical lens SL (using a CMOS2 photodetector). The figure shows theoretical (**a**,**d**) and experimental (**c**,**f**) intensity patterns of structured LG and astigmatic structured LG beams, and a computer simulation of the interference patterns (**b**,**e**) for various states (n, ℓ) and the parameters $\varepsilon=1$; $\theta=\pi/2$ and astigmatic parameter $\beta=\pi/4$.

OAM of beam



The orbital angular momentum was calculated using the formula: $\ell_z=\frac{\sum_{k=0}^{2n+\ell}(2n+\ell-2k)|b_k(\theta,\beta,\varepsilon)|^2(2n+\ell-k)!k!}{\sum_{k=0}^{2n+\ell}|b_k(\theta,\beta,\varepsilon)|^2(2n+\ell-k)!k!}$,

where $|b_k(\theta,\beta,\varepsilon)|^2$ coefficients of modes in beams (2) and n and ℓ are the radial and azimuth number of LG.

When constructing the OAM curve for a structured beam, we found sharp bursts of OAM at certain beam parameters, which were confirmed by experiment. The burst height corresponds to half the sum of the indices of the structured LG beam.

Conclusion

When a structured SLG beam passes through an optical astigmatic element, such as a cylindrical lens or a spherical lens whose axis does not coincide with the axis of the optical system, its internal structure changes radically. Such an astigmatic transformation increases the number of degrees of freedom of structured beams and expands the scope of their technical application. However, in some cases it is necessary that the astigmatic element does not change the structure of the light beam. It is the study of the conditions for the formation of astigmatically invariant structured LG beams that was carried out in our work. We also found sharp OAM spikes and dips in astigmatic structured LG beams in the region where the OAM vanishes. The height and depth of these bursts and dips significantly exceed the maximum and minimum OAM values in ordinary structured LG beams. It was shown that the occurrence of OAM bursts and dips is caused by a radical rearrangement of the modes in the beam in the form due to ordering. Theoretical calculation, accompanied by computer simulation, and experiment are in good agreement with each other.