Study of astigmatic transformation of Legendre polynomial beams

L.V. Khismatullina

Introduction

The need to expand the capabilities of laser beams in various applications leads researchers to search for new types of structured beams. Note, the properties of some polynomials remain unexplored and underestimated. In this paper, the properties of Gaussian beams with a polynomial Legendre distribution and their astigmatic transformation are studied.

Theoretical background

Hermite–Gauss astigmatic beams:

$$\Psi_a(x,y) = H_{n,m}\left(\frac{x}{\sigma},\frac{y}{\sigma}\right)\exp(iaxy)\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right), \quad (1)$$

(x,y) are coordinates in the input plane,

α is the astigmatism parameter,

$H_{n,m}$ is the Hermite polynomial,

σ is the parameter characterizing the effective width.

Gaussian beams with a Legendre polynomial distribution:

$$\Upsilon(x,y) = P_{n,m}\left(\frac{x}{\sigma},\frac{y}{\sigma}\right)\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right), \quad (2)$$

$P_{n,m}$ is a Legendre polynomial described by the equation:

$$P_{n,m}(x,y) = P_n(x)\cdot P_m(y), P_0(x) = 1, \qquad P_1(x) = x,$$

$$P_{n+1}(x) = \frac{2n+1}{n+1}xP_n(x) - \frac{n}{n+1}P_{n-1}(x).$$

Astigmatic Gaussian beams with a polynomial Legendre distribution:

$$\Upsilon_a(x,y) = P_{n,m}\left(\frac{x}{\sigma},\frac{y}{\sigma}\right)\exp(iaxy)\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right), \quad (3)$$

To accelerate the calculation of the Fresnel, transform is used its relationship with Fourier transform algorithm and the fast Fourier transform:

$$U(x,y,z) = -\frac{ik}{2\pi z}\exp(ikz)\,exp\left[\frac{ik}{2z}(x^2+y^2)\right]\Im\left\{U_0(u,v)\exp\left[\frac{ik}{2z}(u^2+v^2)\right]\right\}.$$

Results of modeling

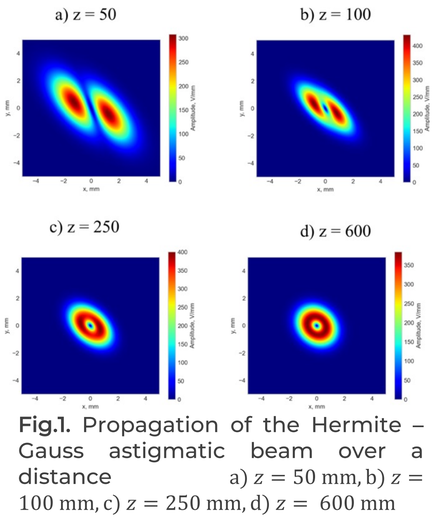

Fig.1. Propagation of the Hermite – Gauss astigmatic beam over a distance a) $z = 50$ mm, b) $z = 100$ mm, c) $z = 250$ mm, d) $z = 600$ mm

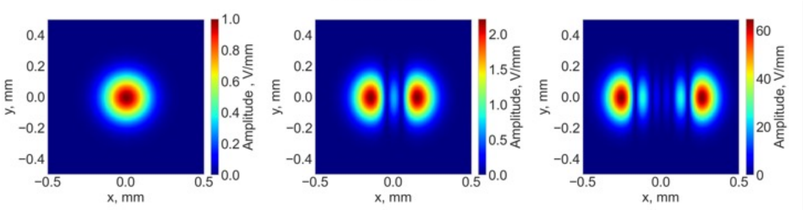

Fig. 2. Modeling of Legendre-Gauss beams (n,m)

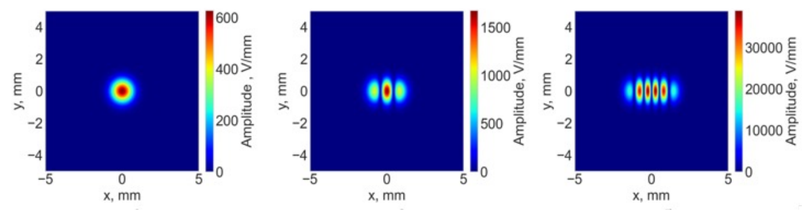

Fig. 3. Modeling of the propagation of Legendre–Gauss beams $(n,0), n = 0,2,5$

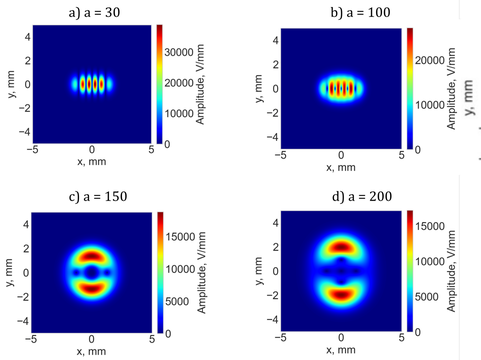

Fig. 4. Amplitude of the Legendre–Gauss astigmatic beam $(3,0)$ with $\alpha = 30\ mm^{-2}, 100\ mm^{-2}, 150\ mm^{-2}, 200\ mm^{-2}$

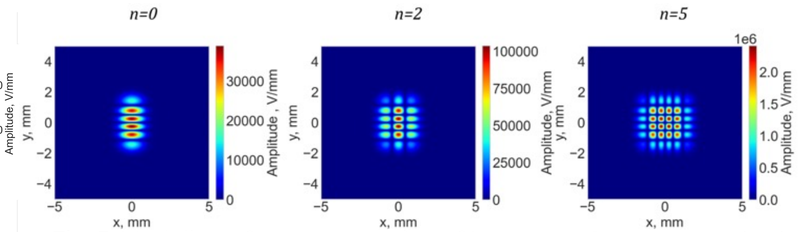

Fig. 5. Modeling of the propagation of Legendre–Gauss beams $(n,5)$

The general effect of increasing the astigmatism parameter is that the Legendre-Gauss beam becomes more elongated in the direction corresponding to the prevailing order and wider in the direction corresponding to the non-prevailing order.

Conclusions

Legendre-Gauss beams and the effect of astigmatic transformation of such beams based on Fresnel transformation are considered and numerically investigated. For a more detailed study of the properties of these beams, certain experimental studies are needed. Based on the numerical results obtained, it can be argued that the studied beams expand the capabilities of laser radiation for some tasks of optical capture and manipulation of micro-objects and material processing.

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

SAMARA NATIONAL RESEARCH UNIVERSITY
34, Moskovskoye shosse, Samara, Russia
www.ssau.ru