Study of the influence of the Henry factor on the dynamics of wide-aperture VCSEL



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Introduction

In this paper, a model describing a wide-aperture laser with a vertical resonator (VCSEL) is analyzed. A linear stability analysis was carried out and dispersion curves were obtained for the system parameters under consideration. Instability regions are also constructed for various values of the Henry factor, which are consistent with the dispersion curves. The stabilizing effects of optical injection are investigated.

Mathematical model

To describe the dynamics of vertical cavity lasers, we use the system of equations [1-3]:

$$\begin{cases} \frac{\partial E}{\partial t} = -[1+i\theta + 2C(i\alpha - 1)(N-1)]E + i\Delta_{\perp}E + E_{inj}, \\ \frac{\partial N}{\partial t} = -\gamma \left[N - I + \left| E \right|^2 (N-1) \right] + \gamma d\Delta_{\perp}N, \end{cases}$$
(1)



Here, E the mean-field dynamics of the complex field amplitude, N carrier density, where θ is the cavity detuning parameter, α is the linewidth enhancement factor of the semiconductor [4], and γ is the carrier decay rate, normalized to the photon relaxation rate. The parameter C represents the interaction between carriers and field, and depends on the laser differential gain and the photon relaxation rate. The pump current, I, generates the carriers within the active region, which diffuse in the transverse direction according to the diffusion factor d. External injection is characterized by E_{inj} denotes the injection strength.

Homogeneous steady-state and linear stability

analysis

The homogeneous solution $(E_{0,}N_{0})$ of equations is readily obtained by setting equal to zero the time derivatives and neglecting the Laplacian operator. What one obtains is

$$\left|E_{inj}\right|^{2} = \left|E_{0}\right|^{2} \left[\left(\theta + 2C\alpha\left(N_{0}-1\right)\right)^{2} + \left(1-2C\left(N_{0}-1\right)\right)^{2}\right], \quad N_{0} = \frac{I+\left|E_{0}\right|^{2}}{\left|E_{0}\right|^{2}+1}$$
(2)

The linear stability of a homogeneous solution is analyzed by studying the response of the system to small perturbations in the vicinity of stationary values. Let's assume that

$$\begin{cases} E = E_0 + \delta E_0 \exp(\lambda t + i(q_x x + q_y y)), \\ E^* = E_0^* + \delta E_0^* \exp(\lambda t + i(q_x x + q_y y)), \\ N = N_0 + \delta N_0 \exp(\lambda t + i(q_x x + q_y y)), \end{cases}$$
Substituting expressions into system , we obtain the following characteristic equation: $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$, (3)
System parameters $\theta = -1.5, C = 0.45, I_0 = 4, d = 0.052, \gamma = 10, \alpha = -5$

The homogeneous steady state will be unstable against spatially modulated perturbations if at least one solution of equation (3) has a positive real part for a nonvanishing value of q. In particular, the boundary instability in the plane (q, E_0) (see figure 1) is characterized by $a_3=0$. According to the Routh–Hurwitz criterion, the unstable domain corresponds to the parameter subspace where $a_3<0$.



The instability domain is defined by two limit values $|E_{\pm}|$, which determines the unstable area Scurve in Figure 2. The intersection of the instability domain with the axis q = 0 corresponds to those values E_0 , which the homogeneous stationary solution is unstable against a plane-wave perturbation. This domain of instability is shown in gray in Figure 1. Accordingly, the unstable region is shown in Figure 2 by a solid curve, and the stable-dotted. You can confirm these results by taking a solution that satisfies the implicit expression (2) and obtain dispersion curves (Figure 3 and 4). As already mentioned, the intersection of the instability domain with the axis q=0 determines those stationary values that are unstable to plane wave perturbations. Taking this into account, it is possible to obtain domains of instability with varying system parameters. For example, in Figure 5, areas of instability in the plane (α , E_0) are shown in gray.

The stabilizing effect of external optical injection can be considered the formation of spatial patterns (figure 6,7) for $\theta = -1.5$, C = 0.6, $I_0 = 1.85$, d = 0.052, $\gamma = 0.1$, $\alpha = -5$



Figure 5. Area of unstable values $|E_0|$, when varying the alpha parameter for q=0

Figure 6. Stationary stripe pattern obtained numerically

Figure 7. Stationary hexagonal pattern obtained numerically

References

to instabilities

Figure 3. Real Λ and imaginary parts

equation for $|E_0|=2.04$, corresponding

 Ω of the roots of the characteristic

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Figure 4. All $\Lambda < 0$

stability.

for $|E_0|=5.16$, corresponding to

The paper deals with the issues of stabilization of the dynamics of a wide-aperture laser using external optical radiation. It is found that the suppression of unstable modes is possible only for stationary values, from the upper branch of a homogeneous stationary solution, while the stationary values of the lower branch fall into the Hopf instability, which will be investigated in the future. The growth of the Henry factor and pumping increases the injection threshold value required for stabilization. The formation of spatial structures, the spatial size of which is consistent with linear analysis, is found.