

# Comparative study of power-law apodizing functions when encoding the wavefront in order to increase the depth of focus

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**Abstract.** In this paper, a comparative study of the effect of high-order polynomial phase apodization of an optical system on increasing the depth of focus is carried out. On the basis of numerical modeling, the dependencies of the conservation of the point scattering function on the parameters of the apodizing function during defocusing are constructed.

**Theoretical background.** The optical imaging system consists of a lens, the transmission function of which has the following form:

$$l(x, y) = e^{-ik\frac{(x^2+y^2)}{2f}},$$

where  $k=2\pi/\lambda$  is the wavenumber of the incident radiation with wavelength  $\lambda$ ,  $f$  is the focal length of the lens.

Consider the phase apodization of the lens in the following form:

$$T(x, y) = \left[ e^{-\frac{x^2}{\sigma^2}} e^{-\frac{y^2}{\sigma^2}} e^{i\alpha x^q} e^{i\alpha y^q} \right] e^{-ik\frac{(x^2+y^2)}{2f}},$$

where  $\sigma$  is the radius of the Gaussian beam,  $q$  is the degree parameter,  $\alpha$  is the scaling parameter.

The calculation of the PSF of an apodized optical system in defocusing conditions can be performed using the Fresnel transformation for the function  $T(x, y)$ :

$$F(\xi, \eta, z) = \iint_{-\infty}^{+\infty} T(x, y) e^{ik\frac{[(x-\xi)^2+(y-\eta)^2]}{2z}} dx dy.$$

To find the normalized standard deviation  $\delta$  for the PSF at different distances from the focal plane  $\Delta = f - z$ , we will use the following formula:

$$\delta = \sqrt{\frac{\sum_{i=0}^n \frac{\sum_{j=0}^n (F_{ij} - O_{ij})^2}{\sum_{j=0}^n O_{ij}^2}},$$

where  $O_{ij}$  are the counts of the ideal field ( $\Delta = 0$  mm),  $F_{ij}$  are the counts of the other fields ( $\Delta \neq 0$  mm).

**Results of modeling.** The following calculation parameters were used for the presented results:  $f = 300$  mm,  $\sigma = 0,5$  mm,  $\lambda = 5 \cdot 10^{-4}$  mm, the distance from the focusing plane  $\Delta$  varied up to 150 mm in both directions.

	$\Delta = -150$ mm	$\Delta = -100$ mm	$\Delta = 0$ mm	$\Delta = 100$ mm	$\Delta = 150$ mm
$q = 3$					
$\delta$	0.9237	0.3444	0	0.1047	0.2159
$q = 5$					
$\delta$	1.0755	0.8049	0	0.4002	0.5404
$q = 7$					
$\delta$	0.9613	0.9178	0	0.5789	0.7304

Fig. 1. PSF patterns with apodization by the function  $T(x, y)$  of the third, fifth and seventh degree at  $\alpha=50$  for different defocusing distances  $\Delta$

Based on the results obtained (Fig. 1), we can say that the best preservation of the PSF pattern during defocusing is demonstrated by cubic ( $q=3$ ) phase apodization – the patterns almost do not change at  $\Delta = \pm 150$  mm. With an increase in the degree ( $q=5;7$ ), the PSF variations are more noticeable.

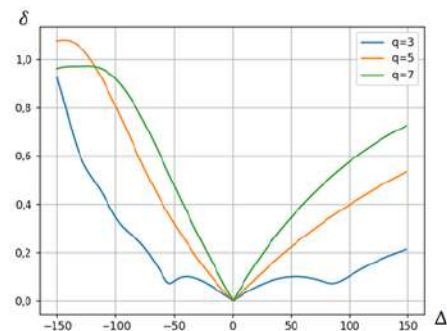


Fig. 2. Graphs of the dependence of the standard deviation on the defocusing value  $\Delta$  for the functions  $T(x, y)$  of the third, fifth and seventh degrees, if  $\alpha=50$

**Conclusions.** The results of this work showed that an increase in the degree of the phase function of apodization leads to a decrease in the region of invariance of the PSF during defocusing, however, at the same time, the PSF patterns become closer to the distribution of the delta function, which provides less distortion of the encoded image.