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APPLICATION OF ANALYTICAL DESIGN OF AGGREGATED REGULATORS METHOD TO NUTRIENT-PHYTOPLANKTON-ZOOPLANKTON MODELS

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Relevance

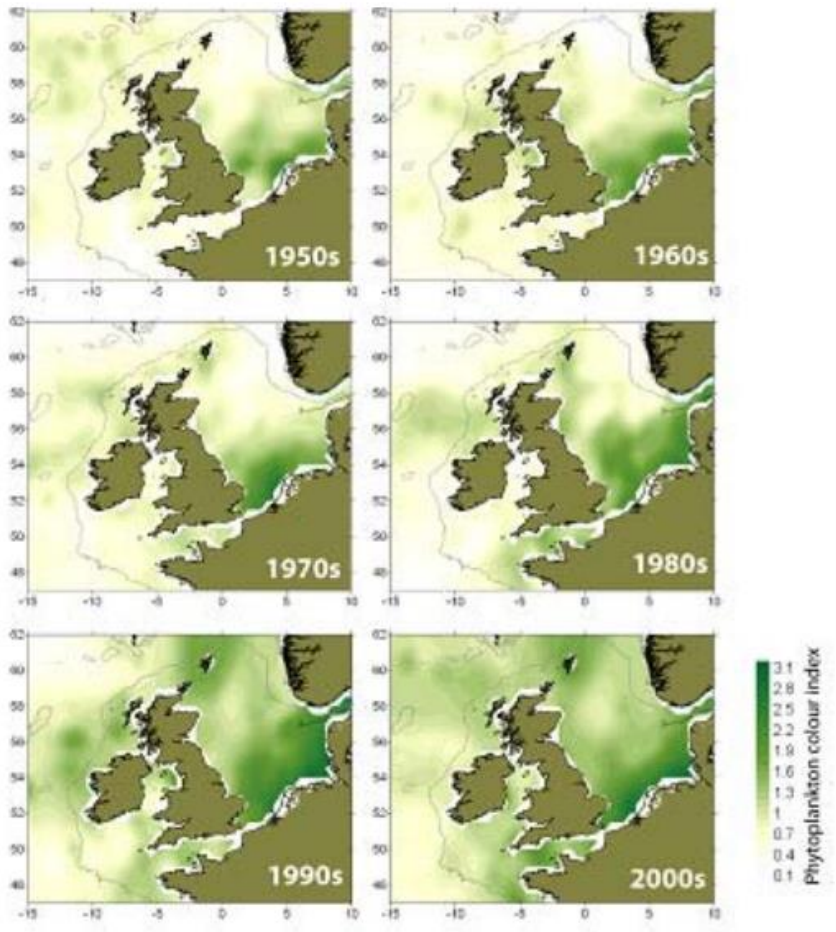

Figure 1. Mean spatial distribution of phytoplankton standing stock (Phytoplankton Colour
Index, or PCI) per decade from the 1950s to the present.*

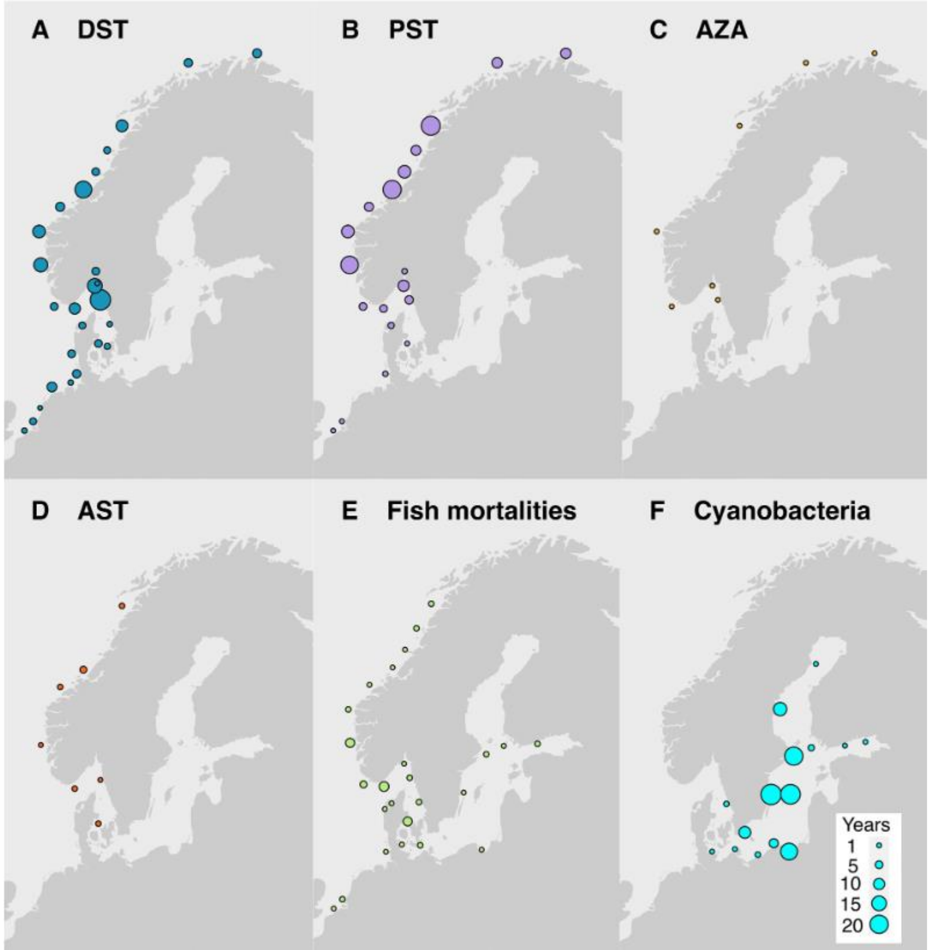

Figure 2. The distribution of harmful algal events along the northern part of European Atlantic during the period 1987 to 2019. Size of circles represents the number of years of reported events**

* Licandro, Priscilla, et al. "Overview of trends in plankton communities." ICES, 2011.

** Karlson, B., Andersen, P., Arneborg, L., Cembella, A., Eikrem, W., John, U., … Suikkanen, S. Harmful algal blooms and their effects in coastal seas of Northern Europe. Harmful Algae. 2021. № 102. Pp. 101989.

Purpose and objectives

Purpose: creation of a new model of algae bloom based on the three-component nutrition-phytoplankton-zooplankton (NPZ) model and synergetic control theory

Objectives:

- Determine the required states of dynamic systems
- Synthesize control for given states of systems
- Determine the criteria for the transition of the system from one stationary state to another
- Using computer simulation to check the theoretical results

Base NPZ-model

$$\begin{cases} \dfrac{dx_1}{dt} = ax_2 + bx_3 - cx_1x_2 \\ \dfrac{dx_2}{dt} = cx_1x_2 - dx_2x_3 - ax_2 \\ \dfrac{dx_3}{dt} = dx_2x_3 - bx_3 \end{cases}$$

x_1, x_2, x_3 densities of nutrition, phytoplankton and zooplankton

$a = 0.05, b = 1, c = 25.003, d = 1.8$*

Stationary points

$E_0(A, 0, 0)$

$E_1(0, A, 0)$

$E_2\left(\dfrac{a}{c}, A - \dfrac{a}{c}, 0\right)$

$E_3(x_1^*, x_2^*, x_3^*)$

$$x_1^* = \frac{a - b + Ad}{c + d}, x_2^* = \frac{b}{d}, x_3^* = \frac{Acd - ad - bc}{d^2 + cd}$$

* González-Parra, G., Arenas, A. J., & Chen-Charpentier, B. M. (2010). Combination of nonstandard schemes and Richardson's extrapolation to improve the numerical solution of population models. Mathematical and Computer Modelling, 52(7-8), 1030–1036. doi:10.1016/j.mcm.2010.03.015

Base NPZ-model

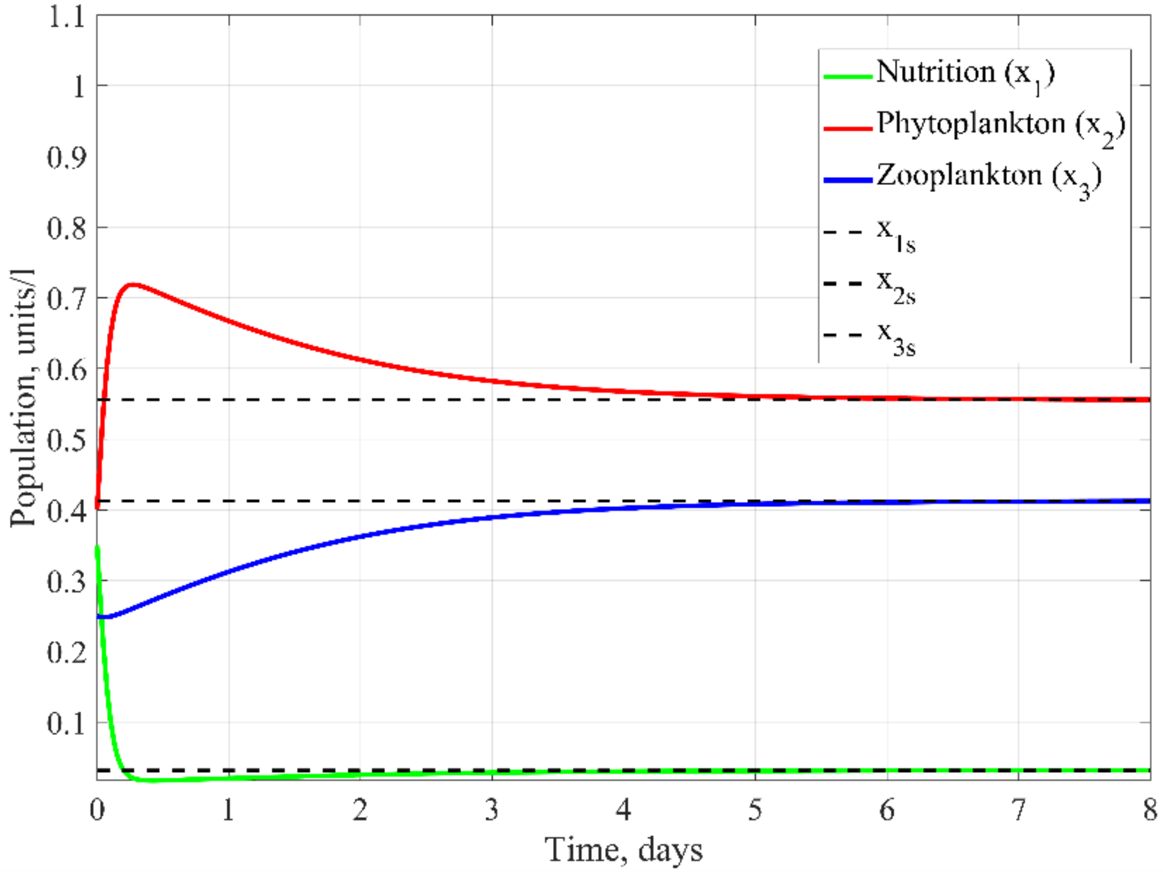

Figure 3. Numerical solution of the NPZ-model

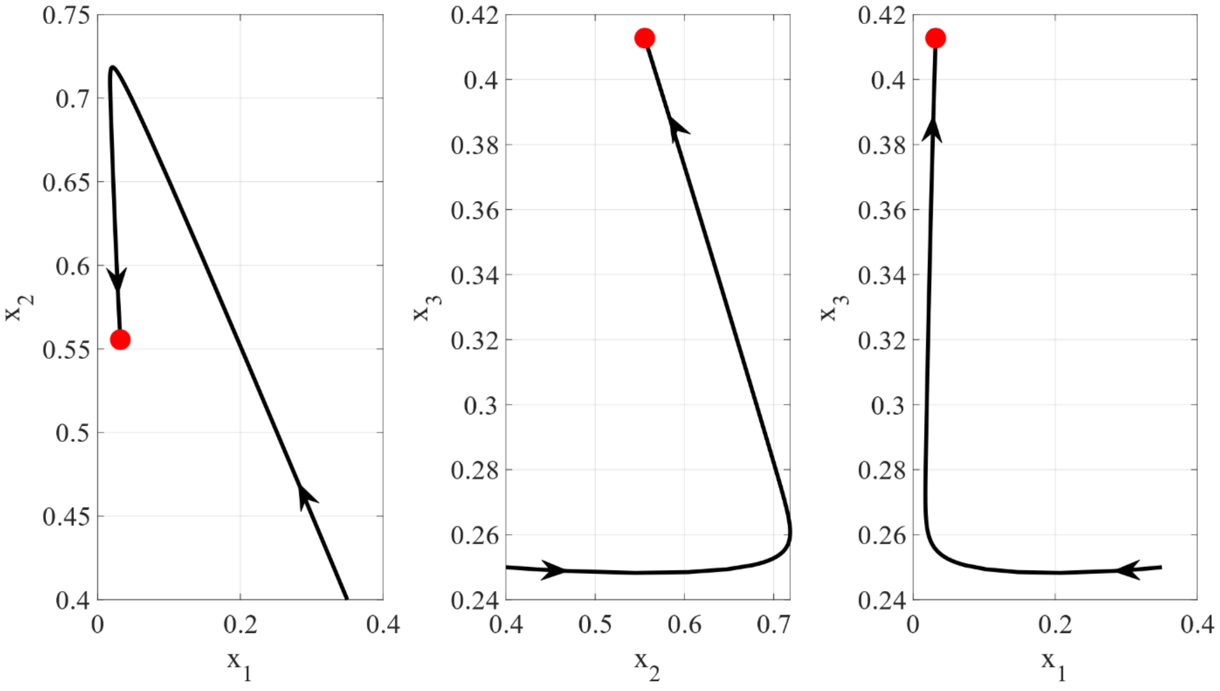

Figure 4. Phase portraits of the solution

NPZ-model with control

$$\begin{cases} \dfrac{dx_1}{dt} = ax_2 + bx_3 - cx_1x_2 + u \\ \dfrac{dx_2}{dt} = cx_1x_2 - dx_2x_3 - ax_2 \\ \dfrac{dx_3}{dt} = dx_2x_3 - bx_3 \\ u = -\dfrac{\psi^{(I)}}{T_1} - ax_2 - bx_3 + cx_1x_2 + \dfrac{d\varphi}{dt} \\ \psi(t) = x_2(t) - x_2^* \\ \psi^{(I)}(t) = x_1(t) - \varphi(t) \\ \varphi(t) = \dfrac{-\dfrac{\psi(t)}{T_2} + dx_2(t)x_3(t) + ax_2(t)}{cx_2(t)} \\ \dfrac{d\varphi(t)}{dt} = \dfrac{\left(-\dfrac{1}{T_2} + dx_3 + a\right)x_2 - \left(-\dfrac{\psi}{T_2} + dx_2x_3 + ax_2\right)}{cx_2^2} f_2 + \dfrac{d}{c} f_3 \end{cases}$$

Stationary points

$E_0\left(x_1^*, x_2^*, 0\right)$ $\quad x_1^* = \dfrac{a}{c}, x_2^*$ - target value

$E_1\left(x_1^*, x_2^*, x_3^*\right)$ $\quad x_1^* = \dfrac{dx_3^* + a}{c}, x_2^*$ - target value

$\psi(t) = x_2(t) - x_2^*$ achieve a given phytoplankton population size

NPZ-model with control

achieve a given phytoplankton population size

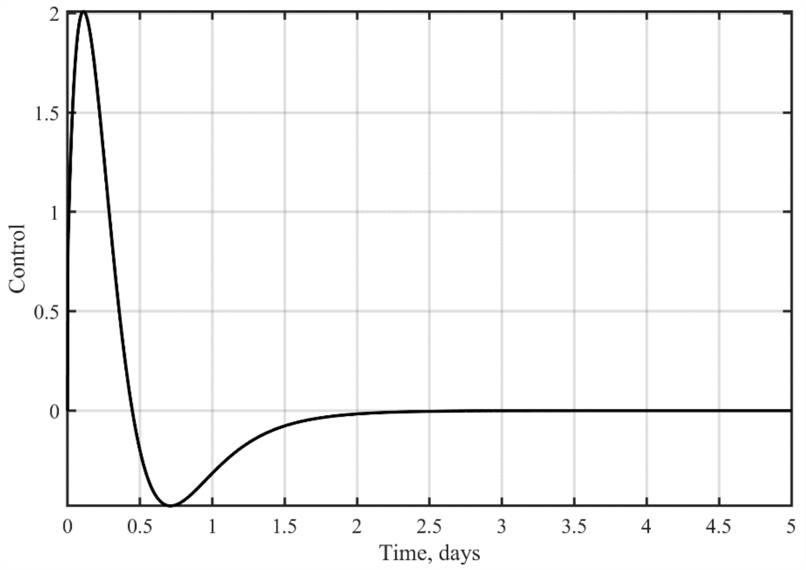

Figure 6. Control function (nutrient flow schedule)

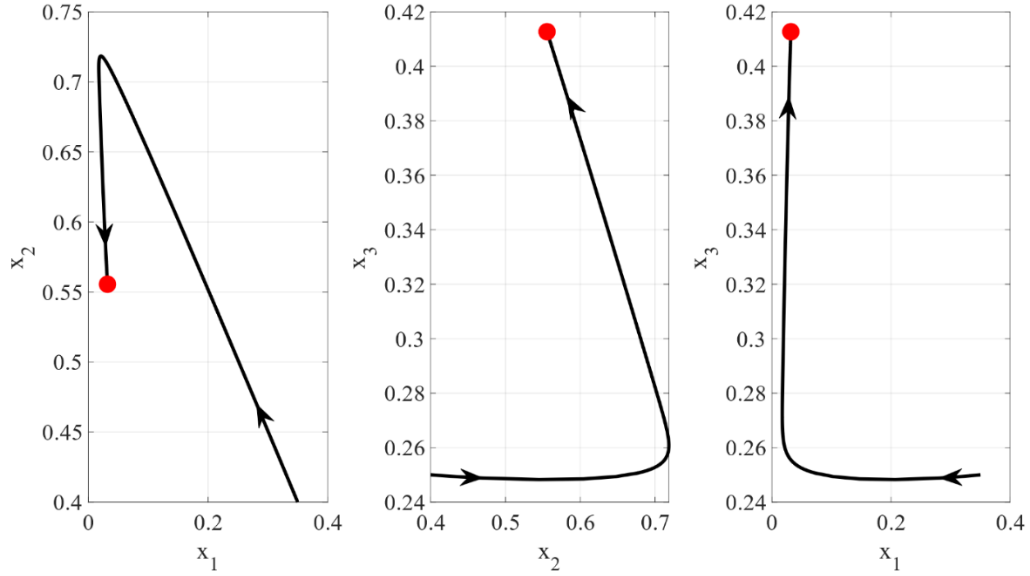

Figure 7. Phase portraits of the solution

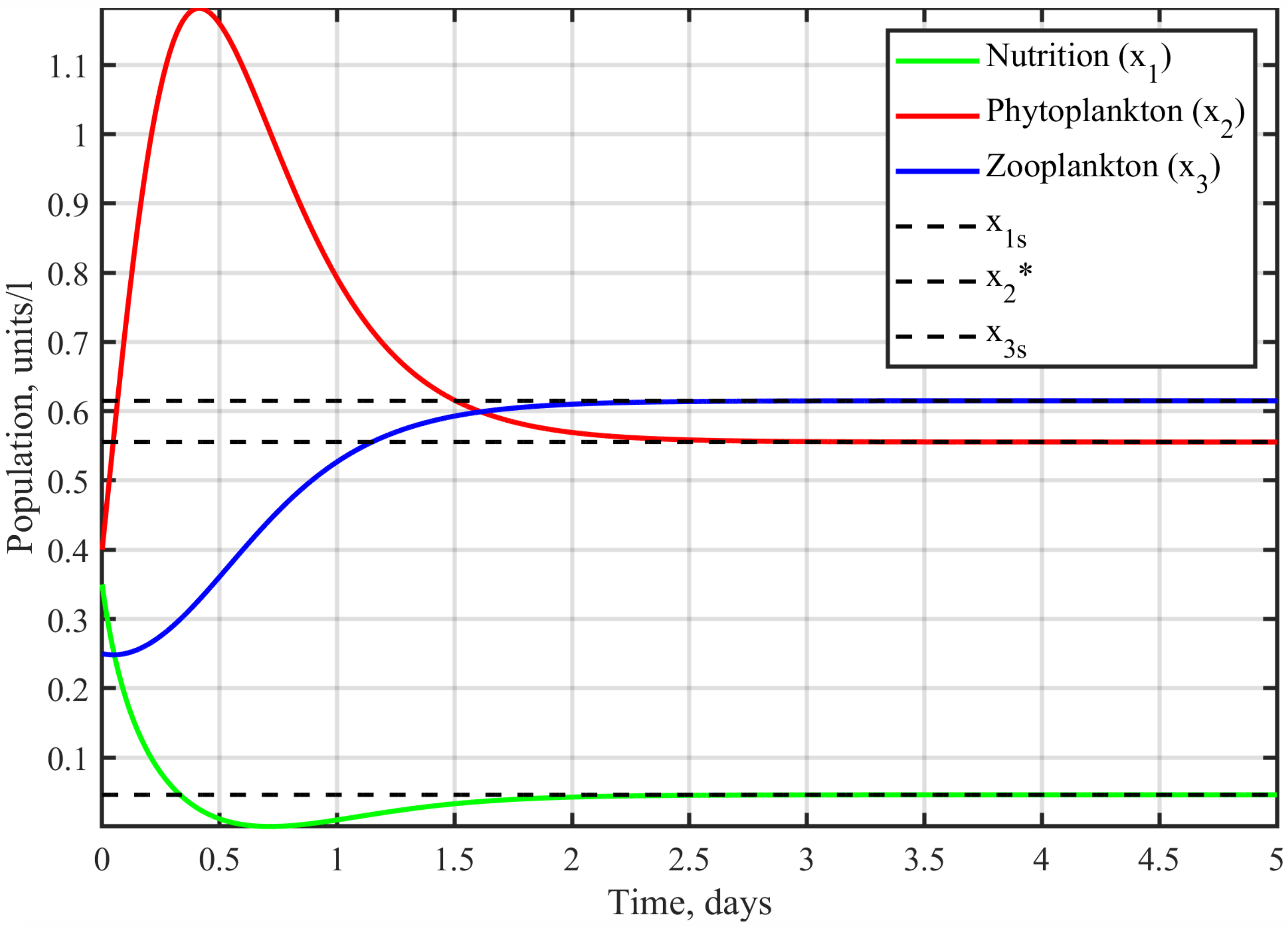

Figure 5. Numerical solution of the NPZ-model

NPZ-model with control

$$\begin{cases} \frac{dx_1}{dt} = ax_2 + bx_3 - cx_1x_2 + u \\ \frac{dx_2}{dt} = cx_1x_2 - dx_2x_3 - ax_2 \\ \frac{dx_3}{dt} = dx_2x_3 - bx_3 \\ u = -\frac{\psi}{\rho T_1} - \frac{f_2}{\rho} - ax_2 - bx_3 + cx_1x_2 \\ \psi(t) = x_2(t) + \rho x_1(t) - q \end{cases}$$

Stationary points

$$E_0\left(x_1^*, 0, 0\right) \quad x_1^* = \frac{q}{\rho}$$

$$E_1\left(x_1^*, x_2^*, 0\right) \quad x_1^* = \frac{a}{c}, x_2^* = \frac{cq - a\rho}{c}$$

$$E_2\left(x_1^*, x_2^*, x_3^*\right)$$

$$x_1^* = \frac{dq - b}{d\rho}, x_2^* = \frac{b}{d}, x_3^* = \frac{cdq - bc - ad\rho}{d^2\rho}$$

$$\psi(t) = x_2(t) + \rho x_1(t) - q$$

achieve proportionality between phytoplankton population and nutrient flow without expanding the phase space

NPZ-model with control

achieve proportionality between phytoplankton population and nutrient flow without expanding the phase space

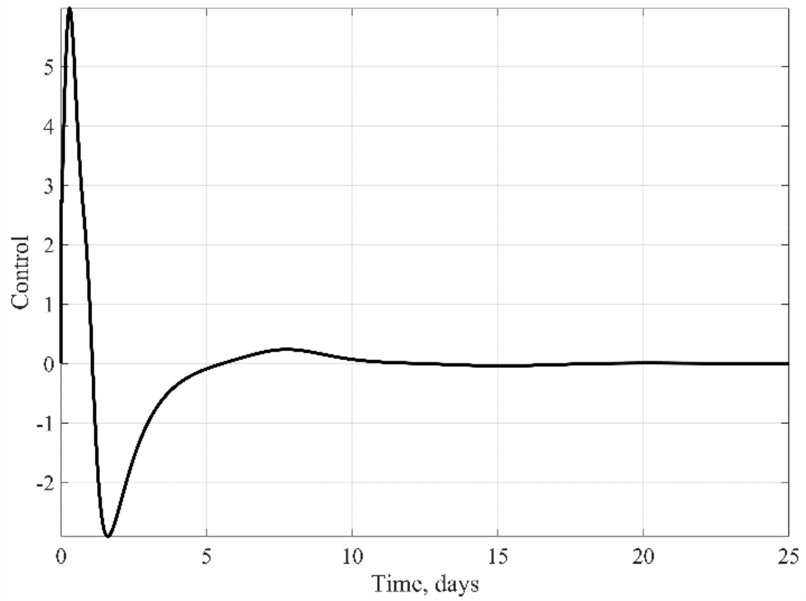

Figure 9. Control function (nutrient flow schedule)

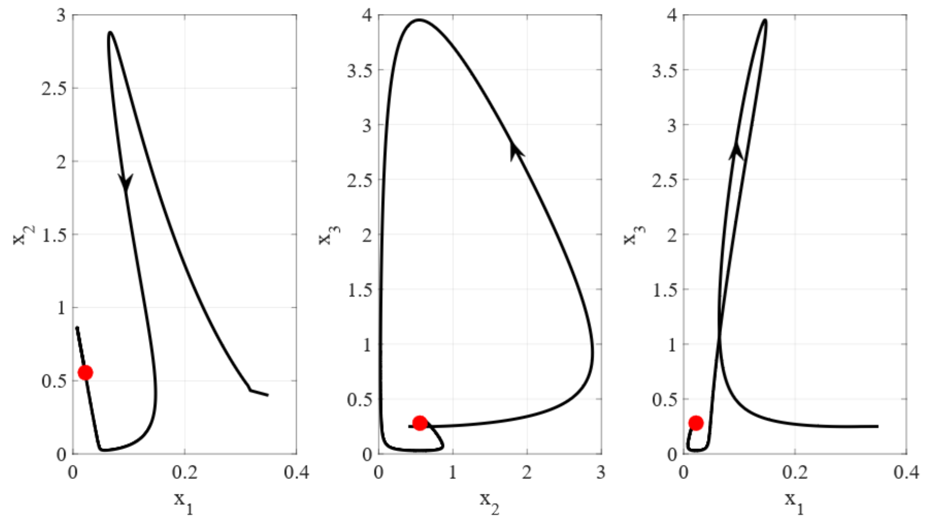

Figure 10. Phase portraits of the solution

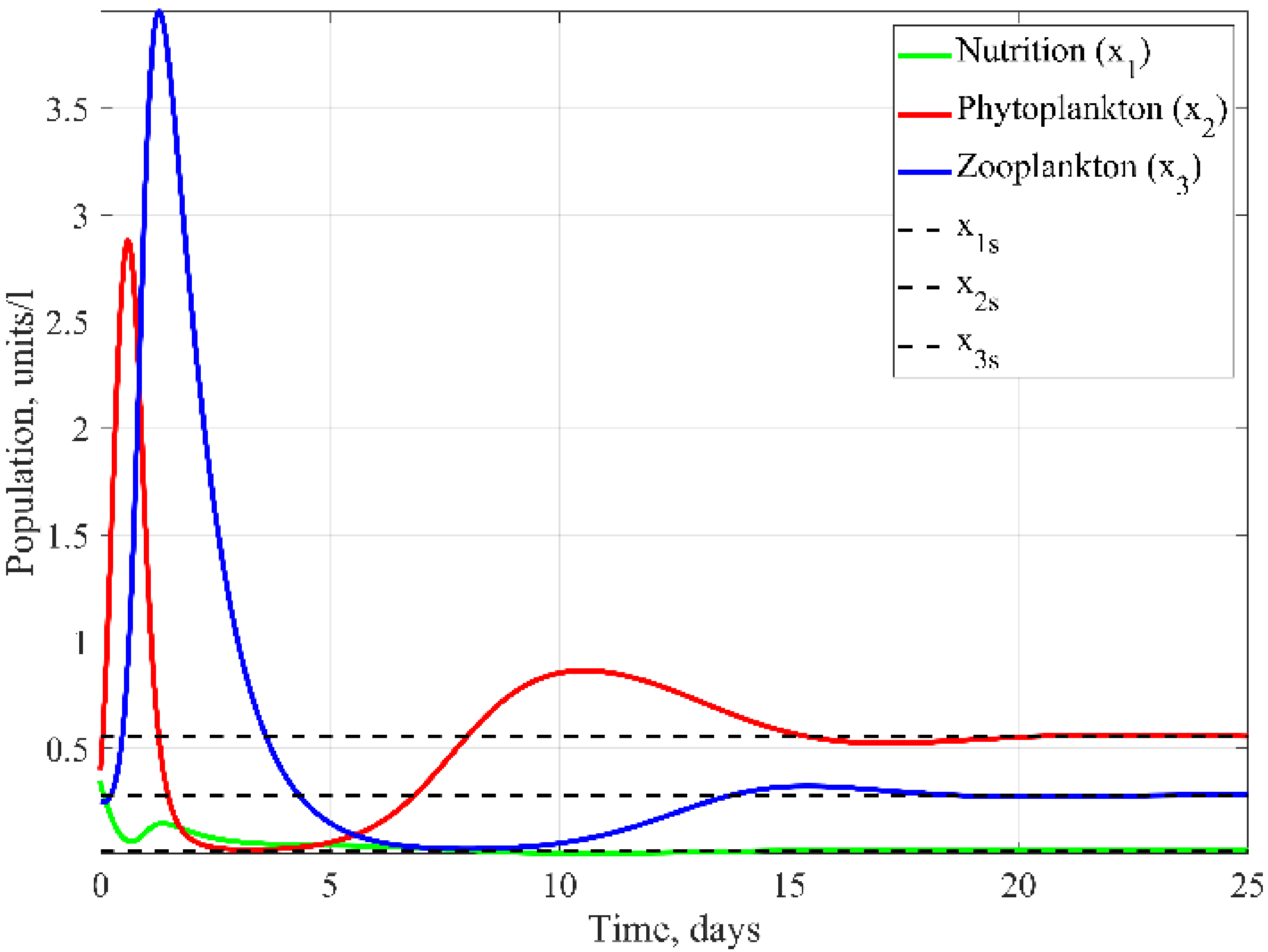

Figure 8. Numerical solution of the NPZ-model

NPZ-model with control

$$\begin{cases}
\frac{dx_1}{dt} = ax_2 + bx_3 - cx_1x_2 + u \\
\frac{dx_2}{dt} = cx_1x_2 - dx_2x_3 - ax_2 \\
\frac{dx_3}{dt} = dx_2x_3 - bx_3 \\
u = -\frac{\psi^{(1)}}{T_1} - ax_2 - bx_3 + cx_1x_2 + \frac{d\varphi(t)}{dt} \\
\frac{d\varphi(t)}{dt} = \frac{\left(-\frac{1}{T_2} + dx_3 + a - \rho dx_3\right)x_2 - \left(-\frac{\psi}{T_2} + dx_2x_3 + ax_2 - \rho(dx_2x_3 - bx_3)\right)}{cx_2^{\,2}} f_2 + \\
+ \frac{-\frac{\rho}{T_2} + dx_2 - \rho(dx_2 - b)}{cx_2} f_3 \\
\varphi(t) = \frac{-\frac{\psi}{T_2} + dx_2x_3 + ax_2 - \rho(dx_2x_3 - bx_3)}{cx_2} \\
\psi(t) = x_2(t) + \rho x_3(t) - q \\
\psi^{(1)}(t) = x_1(t) - \varphi(t)
\end{cases}$$

Stationary points

$$E_0\left(x_1^*, x_2^*, 0\right) \qquad x_1^* = \frac{a}{c}, x_2^* = q$$

$$E_1\left(x_1^*, x_2^*, x_3^*\right)$$

$$x_1^* = \frac{a\rho - b + dq}{c\rho}, x_2^* = \frac{b}{d}, x_3^* = \frac{dq - b}{d\rho}$$

$$\psi(t) = x_2(t) + \rho x_3(t) - q$$

achieve proportional populations of prey and predator
with the expansion of the phase space

NPZ-model with control

achieve proportional populations of prey and predator with the expansion of the phase space

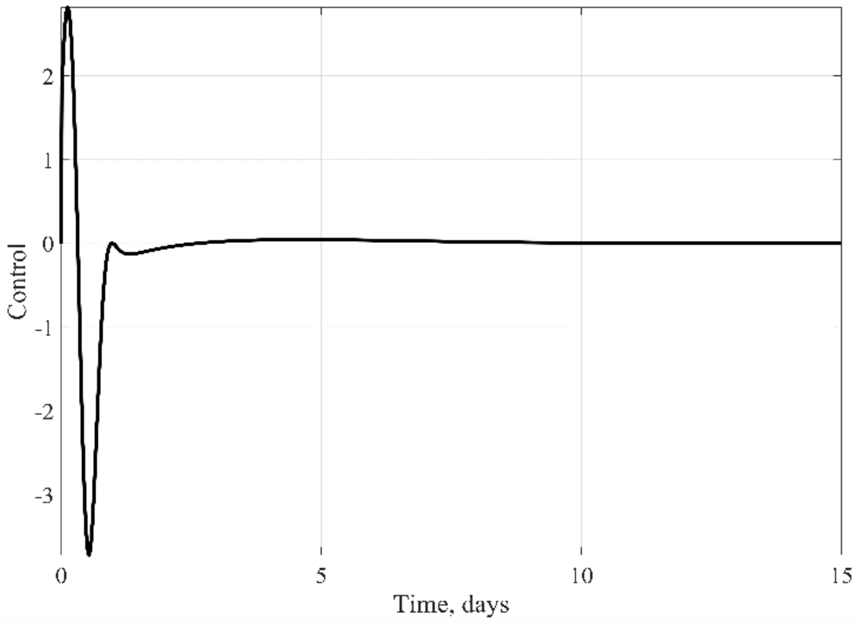

Figure 12. Control function (nutrient flow schedule)

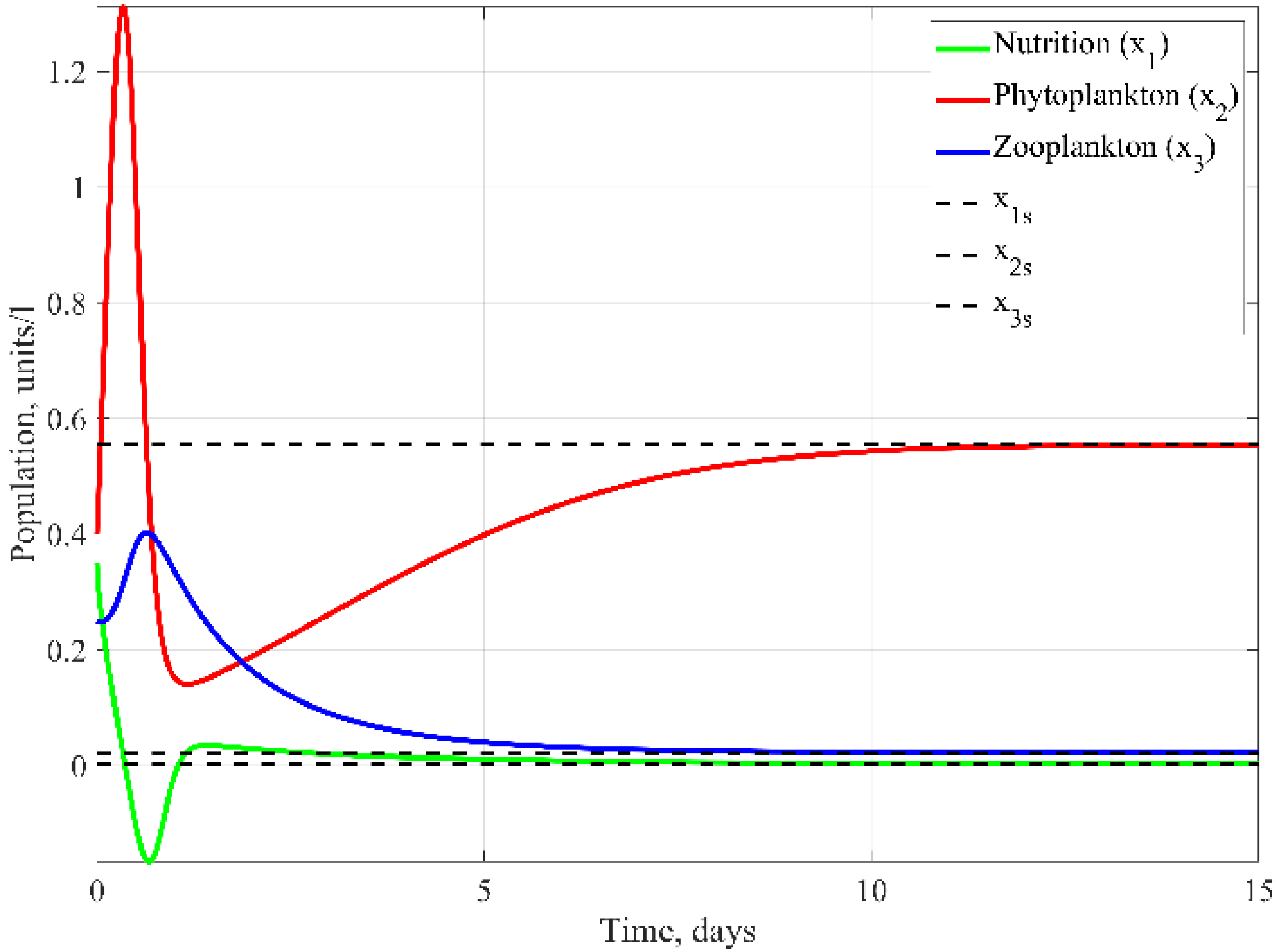

Figure 11. Numerical solution of the NPZ-model

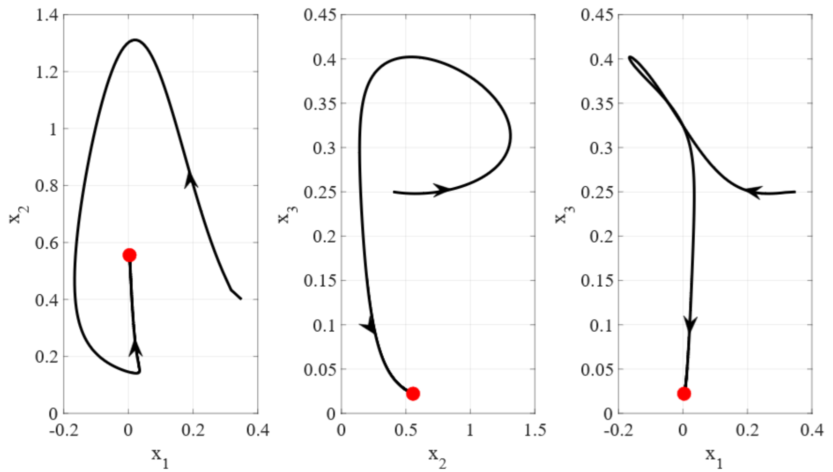

Figure 13. Phase portraits of the solution

Conclusion

1. Control by the ADAR method with different purposes has been synthesized
2. Dynamic systems with new properties and controlled behavior are obtained
3. Numerical solution of models with control is found. The solution is sustainable and has a biological interpretation
4. Analytical expressions for stationary points of the studied systems are found