Algorithm for choosing industrial risk insurance rates based on analysis of large arrays of data on insurance objects

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Methods

We consider an insurance company that has n insured objects. The probability density of the occurrence of an insured event

$$p=e^{-f\xi}$$

depends on the preventive measures carried out by the insurant for the amount of f. The effectiveness of these measures ξ is determined by the results of statistics on the object of insurance for several time periods for a number of parameters. The expected damage

$$X=p\chi$$

depends on the probability p and on the amount of damage χ. The insurance premium V depends on the share of the insured risk γ^S

$$X^S=\gamma^S X,\ 0<\gamma^S<1$$

and the insurance rate T:

$$V=X^S\cdot T,$$

where X^S is the insured damage. This parameter differs from the entire damage, because the insurant can transfer part of the damage γ^S to insurance in order to reduce the insurance premium.

The amount of insurance compensation W depends on the damage $X=p\chi$ and on the level of the insurer's liability α ($0<\alpha\leq 1$):

$$W=X\cdot\alpha.$$

The compensation W depends on the possible damage X and on the insurer's methods of calculating the damage, which are expressed in the parameter α.

The insurer's profit function is

$$\Pi=\sum_{i=1}^{n}(V_i-p_iW_i)=\sum_{i=1}^{n}e^{-\xi_i f_i(T_i)}\chi_i\left(\gamma_i^S T_i-\alpha_i\right) \qquad (1)$$

The problem of searching for the optimal vector of rates $\overrightarrow{T^*}=\{T_1^*,T_2^*,...,T_n^*\}$, is based on the maximization of the insurer's profit

$$\{T_i^*\}=\underset{0<T_i<1}{\arg\max}\,\Pi. \qquad \text{subject to} \quad \begin{cases}\dfrac{df_i}{dT_i}<0,\\ 0<\xi_i<1,\\ \alpha_i\leq\gamma_i^S,\\ \sum_{i=1}^{n}(\gamma_i^S T_i-\alpha_i)>0.\end{cases} \qquad (2)$$

We consider various types of the function $f_i(T_i)$ that satisfy the condition $\dfrac{df_i}{dT_i}<0$. These are the functions of the following form

$$f_i(T_i)=-k_iT_i+b_i, \qquad (3)$$

$$f_i(T_i)=e^{-k_iT_i}+b_i, \qquad (4)$$

$$f_i(T_i)=\frac{k_i}{T_i}+b_i, \qquad (5)$$

where k_i and b_i are the parameters of the function $f_i(T_i)$, they depend on the terms of the insurance contract and on the system of discounts for rates in the frame of the preventive measures.

Results

Proposition: The vector $\vec{T}^*$ is the solution of problem (1)-(2), and it has the following coordinates:

for $f_i(T_i)$ of type (4) T_i^* is solution of equation $$T_i^*+\frac{e^{-k_iT_i^*}}{\xi_ik_i}-\frac{\alpha_i}{\gamma_i^s}=0, \qquad (6)$$

if $k_i\,e^{-k_iT_i^*}(\xi_i-1)(\gamma_i^S T_i^*-\alpha_i)+2\gamma_i^S<0,$

for $f_i(T_i)$ of type (5) $$T_i^*=\frac{\xi_ik_i}{2}\left(\sqrt{1+\frac{4\alpha_i}{\xi_ik_i\gamma_i^S}}-1\right), \qquad (7)$$

if $\dfrac{\xi_i\gamma_i^Sk_i+2\alpha_i}{2}\left[\sqrt{1+\dfrac{4\alpha_i}{\xi_i\gamma_i^Sk_i}}-1\right]-\alpha_i<0$.

FUNCTIONS X, T^* AND PARAMETERS FOR METALLURGICAL AND ELECTRICAL ENTERPRISES

Parameter	**Metallurgical enterprise**	**Electrical enterprise**
ξ	$5.96\cdot10^{-6}$	$1.1\cdot10^{-5}$
χ	555.58 million rubles	39.94 million rubles
X	$555.58e^{-5.96\cdot10^{-6}f}$	$39.94e^{-1.1\cdot10^{-5}f}$
T^*	$T_M^*=39\cdot10^{-5}\left(\sqrt{1+5065.2\frac{\alpha_M}{\gamma_M^S}}-1\right).$	$T_E^*=2.9\cdot10^{-5}\left(\sqrt{1+68610.6\frac{\alpha_E}{\gamma_E^S}}-1\right).$

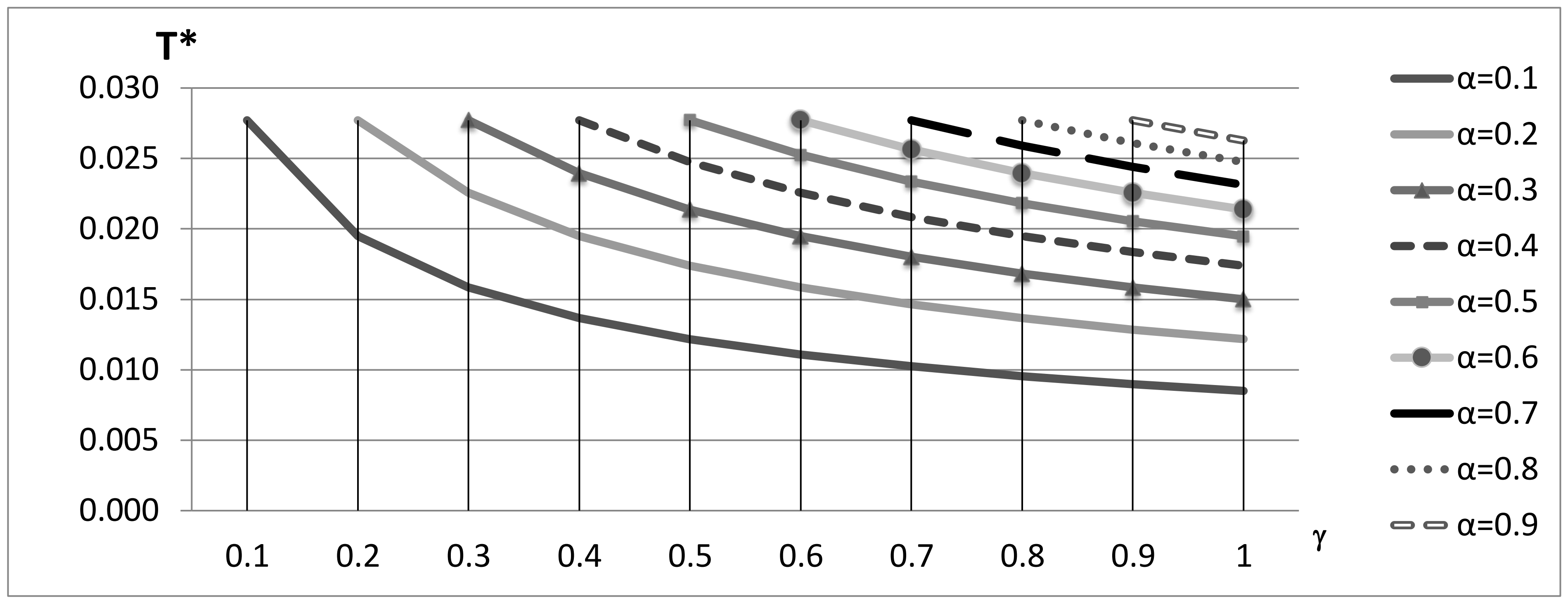

Fig. 1. T_M^* for metallurgical enterprise

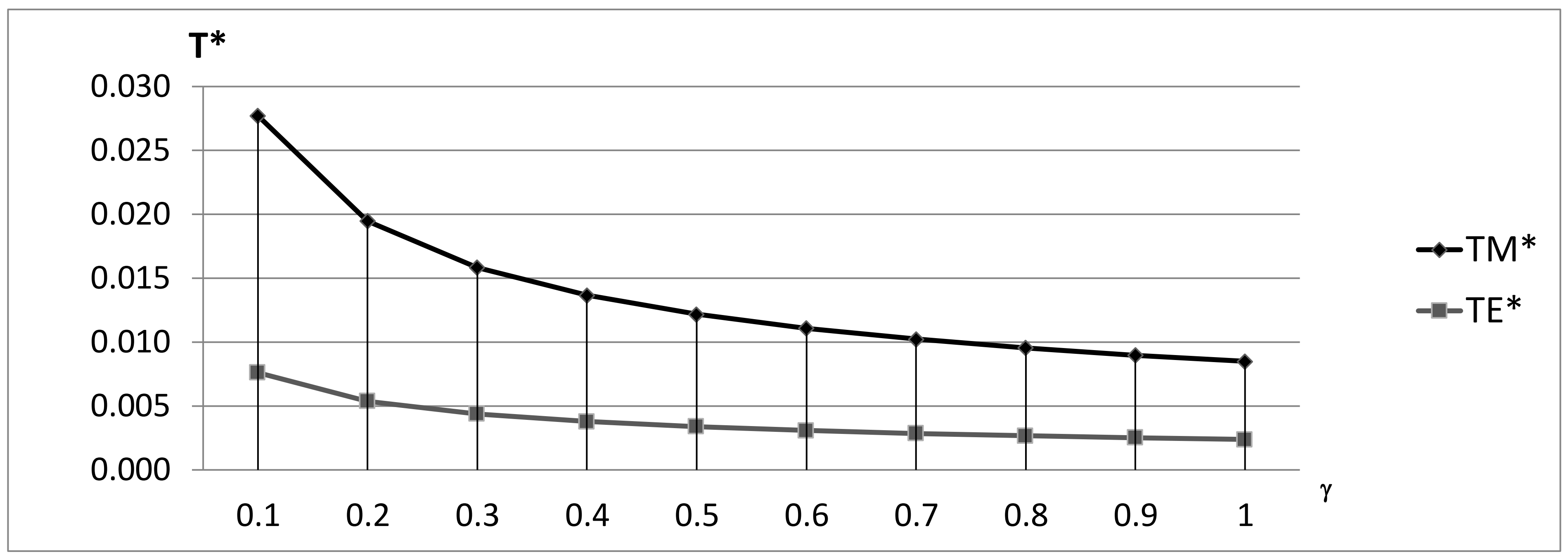

Fig. 2. T^* for metallurgical and electrical enterprises for α=0.1