Methods

We consider an insurance company that has \( n \) insured objects. The probability density of the occurrence of an insured event

\[
p = e^{-\xi}
\]
depends on the preventive measures carried out by the insurant for the amount of \( f \). The effectiveness of these measures \( \xi \) is determined by the results of statistics on the object of insurance for several time periods for a number of parameters. The expected damage

\[
X = pX
\]
depends on the probability \( p \) and on the amount of damage \( X \). The insurance premium \( V \) depends on the share of the insured risk \( \gamma^S \)

\[
V = X^S \gamma^S
\]
and the insurance rate \( T \):

\[
V = X^S \gamma^S, T
\]

where \( X^S \) is the insured damage. This parameter differs from the entire damage, because the insurant can transfer part of the damage \( \gamma^S \) to insurance in order to reduce the insurance premium.

The amount of insurance compensation \( W \) depends on the damage \( X^S \) on the level of the insurer’s liability \( \alpha \) (0 < \( \alpha \) ≤ 1):

\[
W = X^S \alpha.
\]

The compensation \( W \) depends on the possible damage \( X \) and on the insurer’s methods of calculating the damage, which are expressed in the parameter \( \alpha \).

The insurant’s profit function is

\[
\Pi = \sum_{i=1}^{n} (V_i - p_i W_i) = \sum_{i=1}^{n} e^{-\xi_i f_i(T_i)} \chi_i \left( \chi_i^S T_i - \alpha_i \right)
\]

(1)

The problem of searching for the optimal vector of rates \( T^* = (T_1^*, T_2^*, ..., T_n^*) \), is based on the maximization of the insurer’s profit

\[
\{T_i^*\} = \arg \max_{0<T_i<1} \Pi \quad \text{subject to} \quad \begin{cases} \frac{df_i}{dT_i} < 0, \\ 0 < \xi_i < 1, \\ \alpha_i \leq \chi_i^S, \\ \sum_{i=1}^{n} (\chi_i^S T_i - \alpha_i) > 0. \end{cases} \quad (2)
\]

We consider various types of the function \( f_i(T_i) \) that satisfy the condition \( \frac{df_i}{dT_i} < 0 \).

These are the functions of the following form

\[
f_i(T_i) = -k_i T_i + b_i, \quad (3)
\]

\[
f_i(T_i) = e^{-k_i T_i} + b_i, \quad (4)
\]

\[
f_i(T_i) = \frac{k_i}{T_i} + b_i, \quad (5)
\]

where \( k_i \) and \( b_i \) are the parameters of the function \( f_i(T_i) \), they depend on the terms of the insurance contract and on the system of discounts for rates in the frame of the preventive measures.

Results

Proposition: The vector \( T^* \) is the solution of problem (1)-(2), and it has the following coordinates:

for \( f_i(T_i) \) of type (4) \( T_i^* \) is solution of equation

\[
T_i^* + \frac{e^{-k_i T_i^*}}{\xi_i k_i} - \frac{\alpha_i}{\gamma_i^S} = 0, \quad (6)
\]

if \( k_i e^{-k_i T_i^*} (\xi_i - 1)(\gamma_i^S T_i^* - \alpha_i) + 2 \gamma_i^S T_i^* < 0, \)

for \( f_i(T_i) \) of type (5)

\[
T_i^* = \frac{\xi_i k_i}{2} \left( 1 + \frac{4 \alpha_i}{\xi_i \gamma_i^S k_i} - 1 \right), \quad (7)
\]

if \( \frac{\xi_i \gamma_i^S k_i + 2 \alpha_i}{2} \left( 1 + \frac{4 \alpha_i}{\xi_i \gamma_i^S k_i} - 1 \right) - \alpha_i < 0. \)

FUNCTIONS \( X, T^* \) AND PARAMETERS FOR METALLURGICAL AND ELECTRICAL ENTERPRISES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Metallurgical enterprise</th>
<th>Electrical enterprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi )</td>
<td>5.96 \times 10^{-6}</td>
<td>1.1 \times 10^{-3}</td>
</tr>
<tr>
<td>( X )</td>
<td>555.58 million rubles</td>
<td>39.94 million rubles</td>
</tr>
<tr>
<td>( X )</td>
<td>555.58e^{-5.96 \times 10^{-6} f}</td>
<td>39.94e^{-1.1 \times 10^{-5} f}</td>
</tr>
</tbody>
</table>

\[
T^*_M = 39.10^{-5} \left( 1 + 5065.2 \frac{\alpha M}{\gamma M} - 1 \right)
\]

\[
T^*_E = 2.9 \times 10^{-5} \left( 1 + 68610.6 \frac{\alpha E}{\gamma E} - 1 \right)
\]

Fig. 1. \( T^*_M \) for metallurgical enterprise

Fig. 2. \( T^* \) for metallurgical and electrical enterprises for \( \alpha = 0.1 \)