Forecasting the state of a technical object based on a model of a system of quasi-periodic processes in the form of images on a cylinder

V. R. Krasheninnikov*, Yu. E. Kuvayskova and V. N. Klyachkin
*kvrulstu@mail.ru

Introduction

Predicting the technical state of an object is an integral part of ensuring its functional reliability and is aimed at predicting its performance, as well as the onset of a critical state. The more accurately the prediction of the state of the object is carried out, the more timely measures can be taken to eliminate the malfunction that has arisen. Let the technical state of an object at a certain point in time be characterized by a set of diagnosable parameters of its functioning X_1, X_2, …, X_M. Parameter values are recorded at certain points in time and represent a system of time series. However, in the dynamics of the controlled parameters of an object, there is often an irregular periodicity (quasi-periodicity), that is, repetitions with an unpredictability component that cannot be accurately predicted. Therefore, it is necessary to take this feature into account when creating models and methods for processing time series for more accurate forecasting. In this paper, to predict the state of technical objects, it is proposed to use models of systems of quasi-periodic processes in the form of images defined on "multilayer" or "thick-walled" cylinders. Each layer of the cylinder corresponds to one of the time series of the system, the correlations between which are given by the "external" autoregression equation, which allows describing various dependencies between the system processes.

Models of a system of quasi-periodic processes in the form of images

A quasi-periodic process is distinguished by the presence of a double correlation, that is, there is a significant correlation both between adjacent samples and between samples at distances that are multiples of the period. Let us consider a spiral mesh on a cylinder (Fig. 1) to develop a cylinder image model.

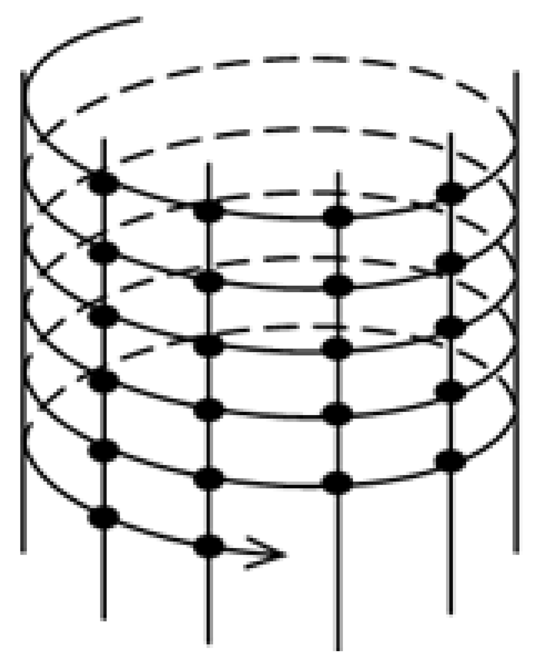

Fig. 1 Grid on a spiral

The rows of this grid are coils of a spiral (helix). Thus, the cylindrical image can be viewed as a single sequence of values along this spiral, which is very convenient for an autoregressive representation.

Let us consider an analogue of the autoregressive Habibi model of a flat image:

$$x_{k,l} = s\, x_{k,l-1} + r\, x_{k-1,l} - s\, r\, x_{k-1,l-1} + q\, \xi_{k,l}$$

where k is the number of the helix turn, l is the node number in the coil, s and r are model parameters, $x_{k,l}$ are independent standard random variables, $x_{k,l} = x_{k+1,l-T}$ for $l \geq T$; T is the period, that is, the number of points in one revolution

The grid in model (1) can be considered in a cylindrical coordinate system, that is, on a system of circles (Fig. 2).

If we represent the model of a cylindrical image as a sweep of the image along a spiral, then the model of a quasi-periodic random process will be obtained:

$$x_n = s\, x_{n-1} + r\, x_{n-T} - s\, r\, x_{n-T-1} + q\, \xi_n$$

where $n = kT + l$.

The value of the parameter s affects the smoothness of the process, that is, the correlation of samples along the lines, and the parameter r affects the correlation of samples at a period distance. At the same time, the values of the parameter r close to 1 show that the neighboring lines of the image (turns of the spiral) are strongly correlated, so this model can be used to. describe the behavior of quasi-periodic time series

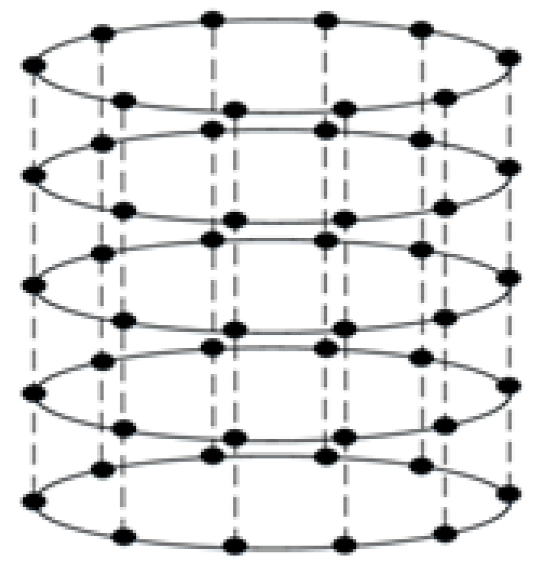

Fig. 2 Grid on a system of circles

Let us consider a model in which each time series of the system is represented by an image on its cylinder, and these individual cylinders are nested into each other and form a system of cylinders, that is, a "multilayer cylinder" or even a rod. Two of such layers are shown in Fig. 3.

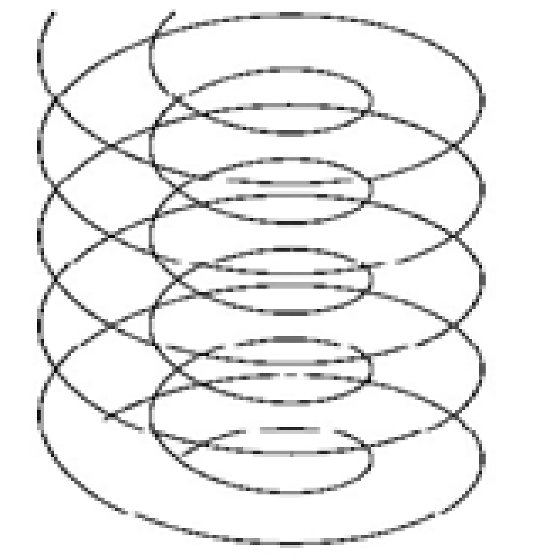

Fig. 3 Grid on a system of two cylinders

Let a system of M homogeneous random processes $X = \{x_{n,m}, m=1,2,\ldots,M\}$, each of which is described using model (2), have a given covariance matrix between processes

$$C = \{C_{ij}\} = \{\mathrm{cov}(x_{n,i}, x_{n,j})\} = \{M[x_{n,i} x_{n,j}],$$
$$i = 1,2,\ldots, M,\ j = 1,2,\ldots, M\}.$$

Let $Y = \{y_{n,m}, m=1,2,\ldots,M\}$ be a system of M independent "standard" processes with zero mean and unit variance, obtained on the basis of model (2), in which the parameter, where $q^2 = q_1^2 = 1/\sigma^2$

$$y_{n,m} = s\, y_{n-1,m} + r\, y_{n-T,m} - s\, r\, y_{n-T-1,m} + q_1\, \xi_{n,m}$$

Then we form the given system X as a linear combination of "standard" processes: $X = AY$, i.e.

$$x_{n,m} = \sum_j a_{mj} y_{n,j},\ m = 1,2,\ldots, M$$

Matrix A must satisfy the equation $AA^T=C$ and can be taken lower triangular.

In particular, if $C_{ij}=\sigma^2 p^{|j-i|}$, that is, processes $x_{n,m}$ have the same variance σ^2 and correlation coefficient $p^{|j-i|}$ between processes $x_{n,i}$ and $x_{n,j}$, then they can be obtained from the standard ones in the form of a first-order autoregression:

$$x_{n,1} = \sigma y_{n,1},\ x_{n,m} = p\, x_{n,m-1} + \sqrt{1-p^2}\, y_{n,m}.$$

Passing from the designations $x_{n,m}$ to the designations $x_{k,l,m}$, we obtain a system of images defined on a system of nested cylinders.

Thus, by setting the values of the model parameters, it is possible to describe systems of quasi-periodic processes with the required correlations within and between periods, as well as between the processes of the system.

Numerical example

Let us consider, as an example of The water purification system at the natural surface water purification and drinking water treatment plant in St. Petersburg (Russia) was taken as an example of the object of study. The technical condition of the water treatment system was assessed in terms of drinking water quality, depending on the physical and chemical parameters of the Western Kronstadt water source.

To assess the effectiveness of the proposed approach, the initial data sample is divided into two parts: training (90% of observations) and control (10% of observations). For the training part, three models of the time series system were built: the integrated moving average autoregression model (ARIMA), the vector autoregression model, and the model of the system of quasi-periodic processes in the form of images on a cylinder. The control part of the sample was used to assess the accuracy of forecasting models. To do this, for all three models, a forecast of the studied system of time series was built and the standard deviation was calculated on the control sample.

The values of standard deviations obtained from the control sample for each row of the system are presented in Table.

Time series	Forecasting accuracy (σ_Δ)		
	ARIMA	*Vector autoregression*	*Model of a system of quasi-periodic processes in the form of images on a cylinder*
X_1	1,75	0,75	0,56
X_2	2,31	1,91	1,86
X_3	6,01	5,85	5,44
X_4	0,19	0,11	0,10
X_5	0,09	0,04	0,03
X_6	1,07	0,68	0,65
X_7	0,21	0,11	0,09
X_8	58,39	56,02	51,37

The obtained results show that the use of a model of a system of quasi-periodic processes in the form of sweeps of several cylindrical images along a spiral when predicting a system of physico-chemical indicators of a water source makes it possible to increase the prediction accuracy up to 3 times compared to the ARIMA model and up to 1.3 times compared to the vector autoregression model.

Acknowledgement

The reported study was funded by RFBR, project number 20-01-00613.

References

1. V.N. Klyachkin, V.R. Krasheninnikov, Yu.E. Kuvayskova, Forecasting and diagnostics of the stability of the functioning of technical objects, RUSAINS, Moscow, 2020. In Russia..
2. V.R. Krasheninnikov, D.V. Kalinov, Yu.G. Pankratov, Spiral Autoregressive Model of a Quasiperiodic Signal, Pattern Recognition and Image Analysis 11 (1) (2001) 211–213.
3. A. Habibi, Two-dimensional Bayesian estimate of images, Proceedings of the IEEE 60 (7) (1972) 878–883. doi: 10.1109/PROC.1972.8787.
4. J. Box, G. Jenkins, Time Series Analysis: Forecasting and Control, Holden–Day, San Francisco, 1970.
5. C.A. Sims, Macroeconomics and Reality, Econometrica 48 (1980) 1–48.

Ulyanovsk State Technical University,
Severny Venets 32, Ulyanovsk, 432027, Russia
www.ulstu.ru

Ульяновский
государственный
технический университет