Investigation of the error of the difference solution of the heat equation in a multilayer medium by the method of a computational experiment

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Introduction

In addition to the classical versions of heat conduction problems for a homogeneous medium, for which the coefficients of the equation are continuous, there are also interesting cases when the medium is inhomogeneous (includes several layers). Such media include multi-layered sheathing of technical structures, biological objects (skin, vessels).

Modeling

To obtain the error, we expand the difference solution for space steps that differ by a factor of two:

$$u_{h_x,h_t} = [u]_{h_x,h_t} + Dh_t + Eh_x + O(h_t^2, h_x^2)$$

$$u_{\frac{h_x}{2},h_t} = [u]_{\frac{h_x}{2},h_t} + Dh_t + E\frac{h_x}{2} + O(h_t^2, \frac{h_x^2}{4})$$

where *u* is the difference solution, [*u*] is the exact analytical solution at the grid nodes, *D,E* are the expansion coefficients.

Subtract one expression from the other. In this case, we will take into account the difference of two analytical solutions will give zero. Thus, we obtain a part of the error $\Delta(h_x)$ associated with the refinement of the step in space:

$$\Delta(h_x) = \left|u_{h_x,h_t} - u_{\frac{h_x}{2},h_t}\right| = E\frac{h_x}{2} + O(h_t^2, \frac{h_x^2}{4})$$

To obtain the part of the error associated with the time step, we will perform similar actions.
As a result, we have the formula:

$$\Delta(h_t) = \left|u_{h_x,h_t} - u_{h_x,\frac{h_t}{2}}\right| = D\frac{h_t}{2} + O(\frac{h_t^2}{4}, h_x^2)$$

By means of a numerical calculation of the error fractions, the expansion coefficients E and D were obtained, which made it possible to establish a formula for the theoretical calculation of the total error:

$$\Delta = 16.79h_x + 12.75h_t$$

Table below shows the deviation ε of the total theoretical error Δ_{theor} calculated by the above formula from the experimental Δ_{exp} obtained by numerical simulation.

h_t	h_x	Δ_{teor}	Δ_{exp}	ε
0.08	0.8	14.59	14.64	0.34%
0.06	0.6	10.95	10.72	1.23%
0.05	0.5	9.12	9.06	0.65%
0.04	0.4	7.29	7.36	0.96%
0.03	0.3	5.47	5.38	1.65%
0.02	0.2	3.65	3.60	1.37%
0.01	0.1	1.82	1.86	2.19%

The table shows that the deviation of the total theoretical error from the experimental one remains small, which makes it possible to make a very accurate prediction. Thus, we can obtain a solution with a given level of error by setting steps.

Conclusion

In the course of the research, a formula was constructed for predicting the error in given steps and it was shown that using this formula it is possible to obtain the desired level of error with a sufficiently high accuracy.

References

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