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Sparse Representation Algorithm in the Problem of Capturing Noise in Images

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Abstract

This article is devoted to the study of image restoration using sparse representation. Sparse representation is a description of an image in the form of coefficients for fragments selected from a predetermined dictionary. This article proposes the author's interpretation of general approach to image restoration using sparse representation and presents results of experiments for one of implementations of this approach.

Existing Approaches

Mathematically, the noise elimination problem can be modeled as following. Let the image be formed as:

$y = x + n,$

where y is the observed noisy image, x is the unknown original image, and n is additive white Gaussian noise.

One of the most common approaches to recovering the original image is Bayesian estimation. This approach can be formulated in the form of a function minimization problem:

$f(x) = \frac{1}{2}\|\hat{x} - y\|_2^2 + G(x),$

where $\hat{x}$ is the estimate of the original image, and $G(x)$ is the smoothness penalty function given by some a priori considerations.

Another significant approach in this paper is based on the application of some filter (usually a low-pass filter) after the Fourier transform, and it can be written as a sequence of transformations:

$\hat{x} = F^{-1}(G(F(y))),$

where $F(.)$ and $F^{-1}(.)$ are the direct and inverse Fourier transforms, respectively.

Sparse Representation

In this study, an algorithm using a sparse representation of images was applied as the basis for development.

The sparse representation describes denoising process in the following form:

$f(x) = \frac{1}{2}\|x - y\|_2^2 + \sum_{j=1}^{K}|a|^p, x = Da,$

where D is a dictionary (matrix) of size $N \times K$. N is the dimension of the signal that we are modeling; K is the dictionary size; a is a vector with a small number of non-zero elements.

Each tile can be represented as a linear combination of several tiles from the redundant dictionary D. The MAP score for denoising this tile for a known dictionary is constructed by solving the expression:

$\hat{a} = \|a\|_0 , \|Da - y\|_2^2 \leq T_0.$

Finding the best dictionary to represent data $\{y_i\}_{i=1}^{N}$ as a sparse representation is done by finding the minimum of the following expression

$\{\|Y - Da\|_F^2\}, \forall\ i\ \|a_i\|_0 \leq T_0.$

Here T_0 is a fixed number of nonzero entries, and the notation $\|a\|_F$ denotes the Euclidean norm.

Proposed Approach

The traditional approach has some drawbacks. Particularly, it requires simultaneous solution of the problem of calculating the sparse representation and minimizing the a priori term.

In article, we propose an approach similar to noise elimination based on the Fourier transform.

Let $J(y, D)$ be a reversible exact sparse representation of an image, i.e. the following condition is satisfied:

$y = J^{-1}(J(y, D), D).$

Let us consider the class of transformations of sparse representations $G(.)$ and formulate the procedure of restoring images in the form

$\hat{x} = J^{-1}(G(J(y, D))).$

As the simplest implementation of this approach, by analogy with the traditional approach, we define function $G(.)$ as a constraint on the maximum number of nonzero coefficients of the sparse representation. Thus, T_0 of the largest coefficients remains unchanged, and the rest are reset to zero.

PSNR Values

Method	σ		
	15	25	50
Sparse Representation	34.78	33.39	31.42
Fourier transform	28.59	27.89	25.52
Bayesian Estimation	26.12	24.06	22.79

Recovery Result

(a)

(b)

(c)

(d)

Examples of research results are shown in Figure: (a) noisy image, (b) reconstructed image using sparse representation, (c) image restored using Fourier transform, (d) image restored using Bayesian approach.

Discussion and Conclusion

In the article we considered the influence of degree of noise in the original image on the restoration result obtained using the sparse representation

It was shown that the proposed method retains the advantages of the sparse representation. However, it simplifies the calculation process due to decomposition and allows more flexible setting of restrictions on the parameters of the sparse representation due to the internal filter function.

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