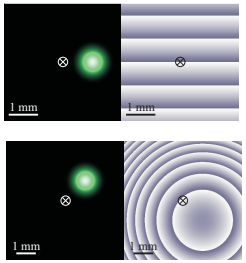
Off-axial propagation-invariant elliptic beams and their orbital angular momentum

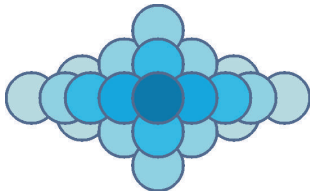
V.V. Kotlyar, A.A. Kovalev

Theory

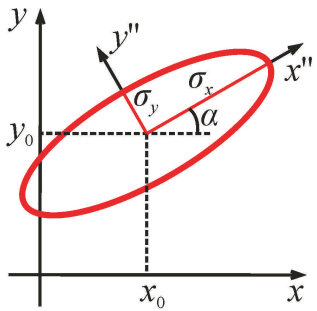


Elementary spiral beam

Abramochkin, E.G.; Volostnikov, V.G. J. Opt. A: Pure Appl. Opt. 2004, 6,



Continuous superposition of the elementary spiral beams on an ellipse



Geometrical parameters of the elliptic light spot

Solution (complex amplitude)

$$E(x,y,z)=\frac{w_0}{w}\exp\left[-\left(\frac{1}{w^2}-\frac{ik}{2R}-\frac{\mu}{w^2}e^{-2i\zeta}\right)x^2-\left(\frac{1}{w^2}-\frac{ik}{2R}+\frac{\mu}{w^2}e^{-2i\zeta}\right)y^2+2i\frac{\mu}{w^2}e^{-2i\zeta}xy\right]$$
$$\times\exp\left\{-2\left(\frac{1}{w^2}-\frac{ik}{2R}\right)\left[x_0(\mu-1)+iy_0(\mu+1)\right](x+iy)-i\zeta\right\}.$$

where $\mu=\gamma e^{-2i\alpha}$, $\gamma=1-\frac{w_0^2}{\sigma_x^2}=\frac{w_0^2}{\sigma_y^2}-1$ and ζ is the Gouy phase

(x_0,y_0) – ellipse center

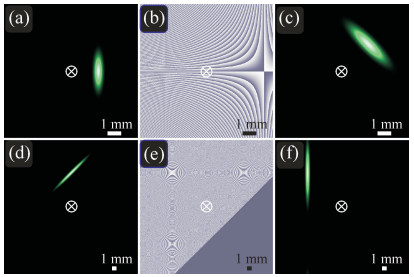
σ_x, σ_y – radii

α – orientation (tilt to the axis x)

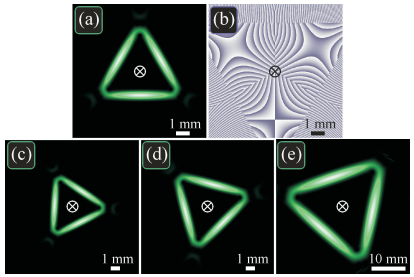
Condition for the ellipse radii:

$$\frac{1}{\sigma_x^2}+\frac{1}{\sigma_y^2}=\frac{2}{w_0^2}$$

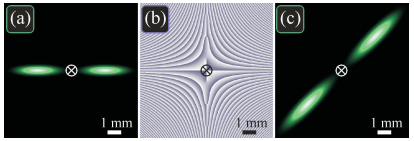
Simulation



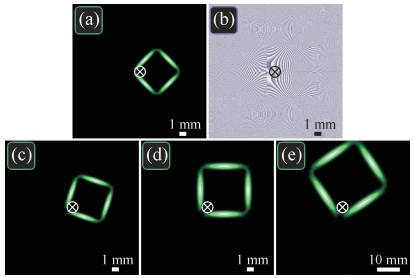
Rotating propagation-invariant ellipse



Rotating propagation-invariant triangle



Rotating propagation-invariant two-petal beams



Rotating propagation-invariant square

Conclusion

We obtained obtained an analytical expression describing monochromatic paraxial propagation-invariant elliptic Gaussian beams with a transverse shift from the optical axis. On free-space propagation, such beam is rotated, but around the optical axis, rather than around its center. We also derived a formula for the orbital angular momentum of such beams. Similarly to the Steiner theorem in mechanics, it is a sum of two terms. One of them describes the intrinsic OAM relative to the 'mass center' (center of the ellipse) and increases with the beam ellipticity. The second term is proportional to the squared distance from the ellipse center to the optical axis. It turns out that the ellipse orientation (tilt angle) in the transverse plane does not affect the normalized orbital angular momentum.

ИСОИ IPSI
IMAGE PROCESSING SYSTEMS INSTITUTE -
BRANCH OF THE FEDERAL SCIENTIFIC RESEARCH CENTRE
“CRYSTALLOGRAPHY AND PHOTONICS” OF RAS
151, Molodogvardeyskaya St., Samara, Russia (www.ipsi.smr.ru)

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY
SAMARA NATIONAL RESEARCH UNIVERSITY
34, Moskovskoe shosse, Samara, Russia (www.ssau.ru)