Off-axial propagation-invariant elliptic beams and their orbital angular momentum

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Theory

Continuous superposition of the elementary spiral beams on an ellipse

Solution (complex amplitude):

\[
E(x, y, z) = \frac{w_0}{w} \exp \left[ \left( -\frac{ik}{2R} \frac{\mu}{w^2} e^{-i\zeta} \right) x^2 - \left( \frac{1}{w^2} - \frac{\mu}{w^2} \frac{e^{-i\zeta}}{w^2} \right) y^2 + 2i \frac{\mu}{w^2} e^{-i\zeta} xy \right] \times \exp \left[ -\frac{1}{w^2} x^2 \left( \frac{1}{w^2} + \frac{\mu}{w^2} \frac{e^{-i\zeta}}{w^2} \right) \right]
\]

where \( \mu = \gamma e^{-2\alpha} \), \( \gamma = 1 - \frac{w_0^2}{\sigma_x^2} - \frac{w_0^2}{\sigma_y^2} \), and \( \zeta \) is the Gouy phase.

Condition for the ellipse radii:

\[
\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2} = \frac{2}{w_0^2}
\]

Simulation

Conclusion

We obtained an analytical expression describing monochromatic paraxial propagation-invariant elliptic Gaussian beams with a transverse shift from the optical axis. On free-space propagation, such beam is rotated, but around the optical axis, rather than around its center. We also derived a formula for the orbital angular momentum of such beams. Similarly to the Steiner theorem in mechanics, it is a sum of two terms. One of them describes the intrinsic OAM relative to the 'mass center' (center of the ellipse) and increases with the beam ellipticity. The second term is proportional to the squared distance from the ellipse center to the optical axis. It turns out that the ellipse orientation (tilt angle) in the transverse plane does not affect the normalized orbital angular momentum.