Calculation of a phase diffractive optical element that forms a given set of spheroidal functions

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Introduction

One of the areas of application of spheroidal functions is the increase in resolution and image recovery [1]. When an image is obtained by optical systems with a finite pupil, it is distorted and information is lost due to the truncation of the spectrum. To restore the signal, the method of analytical continuation of the spectrum can be applied [1]. In this case, the expansion of the know part of the spectrum into various functional series is used, including the spheroidal functions set.
Another approach to transmitting information through finite optical systems without distortion and energy loss is the formation of a signal based on the superposition of spheroidal functions [2].
Spheroidal functions do not have an analytical representation, so they have to be calculated numerically. One way to calculate spheroidal functions, including generalized ones, is to find the eigenfunctions of the corresponding optical operator.
In this paper, a phase diffractive optical element is calculated to form a given set of spheroidal functions. In order to perform calculations, the iterative algorithms are used. There are the Gerchberg-Saxton algorithm, adaptive-additive and adaptive-regularization algorithm.

Modelling

The formed optical distribution $\tau(x,y)$ is defined as follows: $\tau(x,y) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} C_{nm}^{0}\Psi_{nm}(x,y)$, (1) where $N \times M$ is the number of two-dimensional spheroidal functions taken into consideration, $\Psi_{nm}(x,y)$ are spheroidal functions, C_{nm}^{0} are coefficients at the functions.
It should be noted that two-dimensional spheroidal functions can be obtained by the product of the corresponding one-dimensional spheroidal functions.
In the general case, the coefficients C_{nm}^{0} can be arbitrary non-negative values, however, we will use two values: 0 and 1 (Fig. 1). The coefficients C_{nm}^{0} can also be considered as an expansion of the field $\tau(x,y)$ in terms of spheroidal functions.

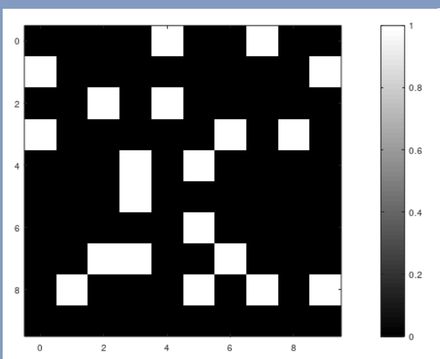

Fig. 1. Absolute values of coefficients of the formed superposition.

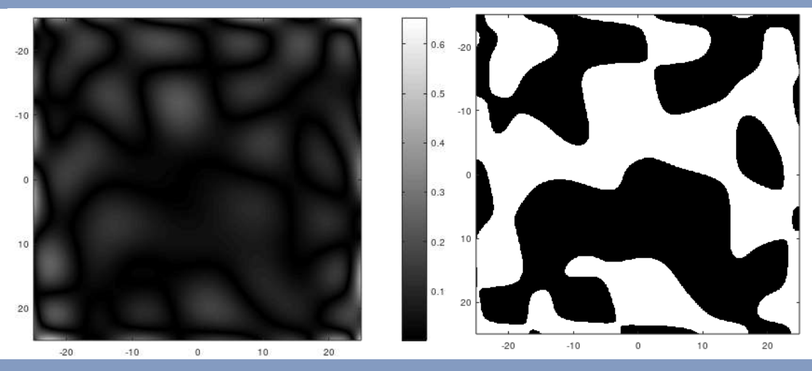

Fig. 2. Initial distribution $\tau(x,y)$.

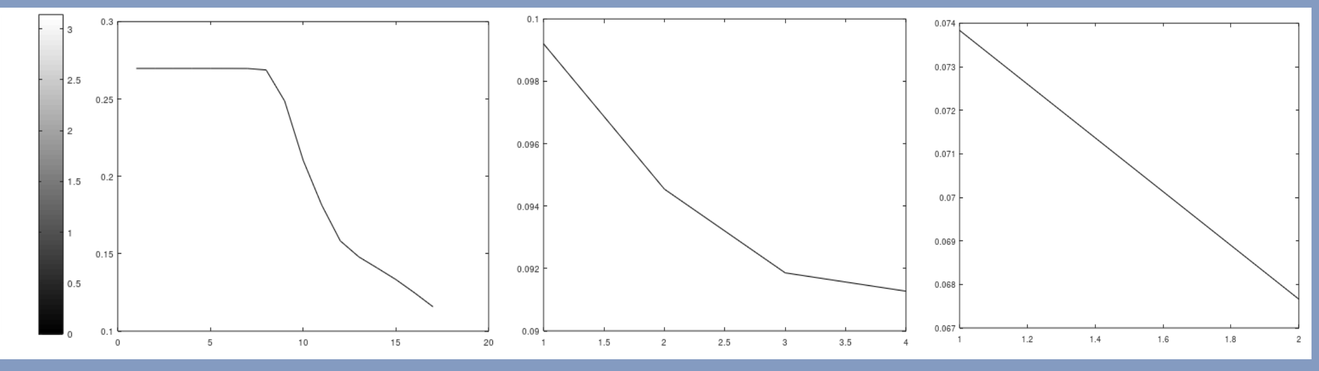

Fig. 3.The graph of RMS change depending on the iteration number of GS-algorithm, AA-algorithm and AR-algorithm.

Our task is to calculate the phase DOE, which, using a lens, forms the distribution (1) shown in Figure 2, the expansion coefficients of which are shown in Figure 1.
Let's run this algorithm and look at the results of its work. The algorithm performed 17 iterations, after which the RMS value stopped decreasing. The RMS chart is shown in Figure 3 (left). The final RMS is 11.57%. Figure 4 (left) shows the values of the expansion coefficients C_{nm}, obtained after the algorithm. The Figure 3 (center) shows the RMS graph after applying the AA-algorithm. The result RMS is 9.12%. Figure 4 (center) shows the values of the expansion coefficients C_{nm}, obtained after the work of the adaptive-additive algorithm. Due to the fact that only 4 iterations have performed, the coefficients have changed slightly compared to the Figure 4 (left), however, visual differences can be found. The Figure 3 (right) shows the RMS graph. Note again that RMS is calculated only for non-zero coefficients. The last algorithm performed only two iterations, but made noticeable improvements in the RMS up to 6.77%. The Figure 4 (right) shows the values of the expansion coefficients C_{nm}, obtained after the work of the adaptive-regularization algorithm. The Figure 5 shows the phase of the DOE that was calculated.
To evaluate the obtained results, we will use the properties of spheroidal functions. As is known, spheroidal functions are eigenfunctions of the Fourier transform. This means that spheroidal features pass through the lens system with minimal change. The Figure 6 (left) shows the expansion coefficients calculated after the passage of the formed superposition through the lens system. The Figure 6 (right) shows each coefficient, which is the result of the product of the formed superposition coefficient and the eigenvalue of the corresponding function. The figures have minor differences, which demonstrate the correctness of the superposition formed with the help of the phase DOE. You can see that about three quarters of the coefficients in both images have gone to zero (or very close to it). This is because the spheroidal functions under consideration have a small number of significant eigenvalues. Figure 7 shows the absolute values of the first ten eigenvalues for one-dimensional spheroidal functions. The values of other eigenvalues are close to zero.

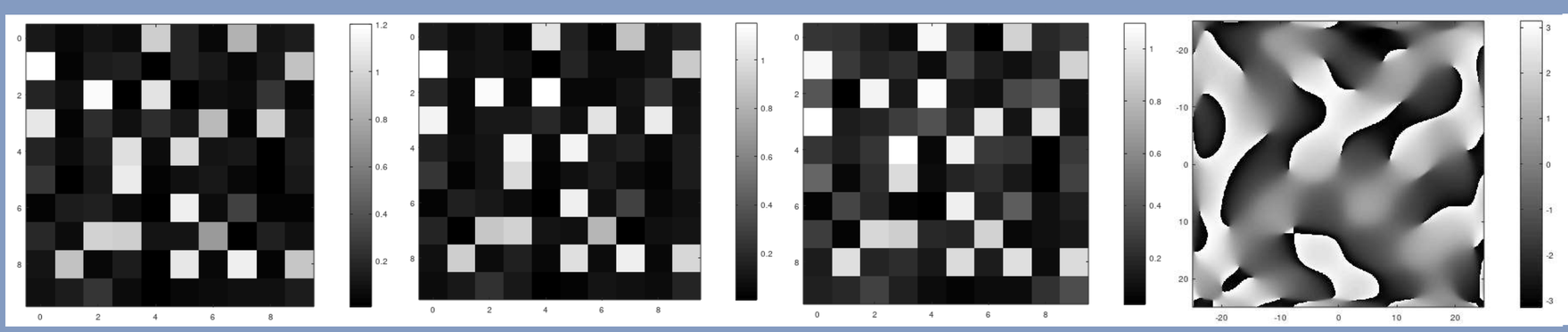

Fig. 4. The absolute values of the expansion coefficients of the phase function $\hat{\tau}(x,y)$, calculated using the GS-algorithm, AA-algorithm and AR-algorithm.

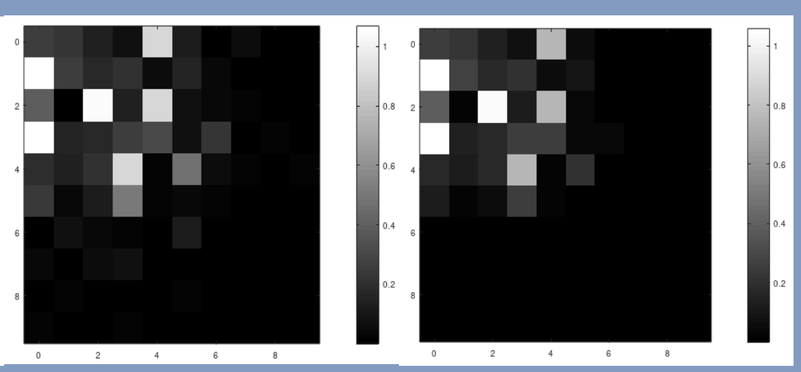

Fig. 5. The transmission function of the calculated phase DOE.

Fig. 6. The absolute values of the coefficients obtained by expanding the field that has passed through the Fourier transform and the absolute values of the coefficients obtained by multiplication by eigenvalues.

Conclusion

In this work, the phase DOE, which forms a superposition of spheroidal functions, was calculated using iterative algorithms. The study showed that after the superposition propagates through the lens system, the expansion coefficients change as expected by multiplying by the eigenvalue (Fig. 7). The deviation of the formed image from the desired one after the successive application of the three algorithms was 6.77%.

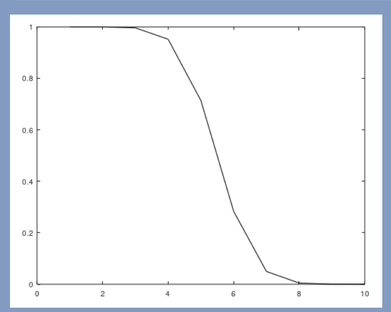

Fig. 7. The absolute values of the first ten eigenvalues of one-dimensional spheroidal functions.

Acknowledgement

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References

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[2] S. N. Khonina and V. V. Kotlyar, “Generating light fields matched to the spheroidal wave-function basis,” Optical Memory and Neural Networks (Allerton Press), vol. 10, no. 4, pp. 267–276, 2001.

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